University of Amsterdam
Faculty of Social and Behavioural Sciences

One-stage meta-analytic structural equation modeling

AERA Systematic Review and Meta-Analysis SIG

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NWO

Content

1. What is (MA)SEM?
2. Univariate versus multivariate methods for MASEM
3. Simulation study results
4. Discussion

What is SEM?

Confirmatory technique to fit hypothesized models to data

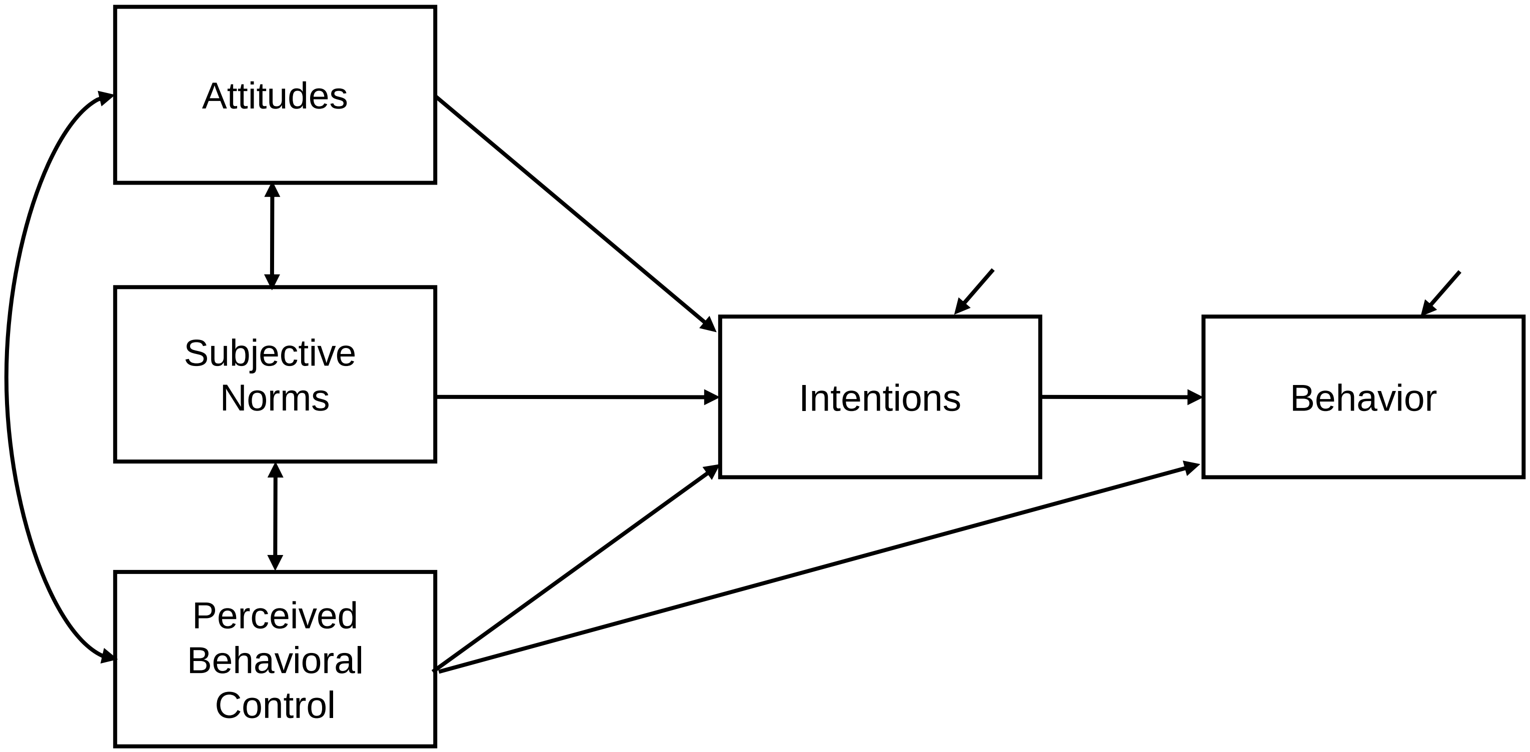
Evaluate indirect effects, relations between latent and observed variables, use fit-indices to compare competing models

E.g. Factor models, path models, full SEM

No raw data needed (covariance matrix and sample size are sufficient)

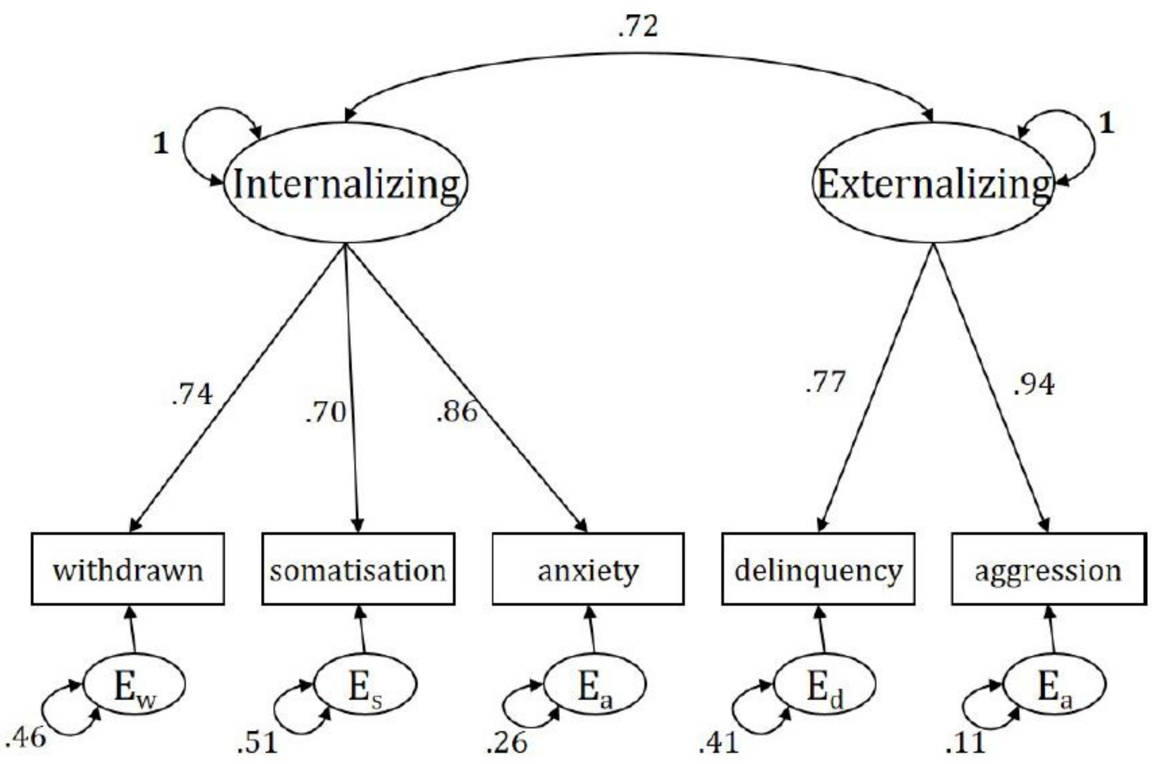
What is SEM?

Example of a path model: Theory of planned behavior from Ajzen and Fishbein (1980)



What is SEM?

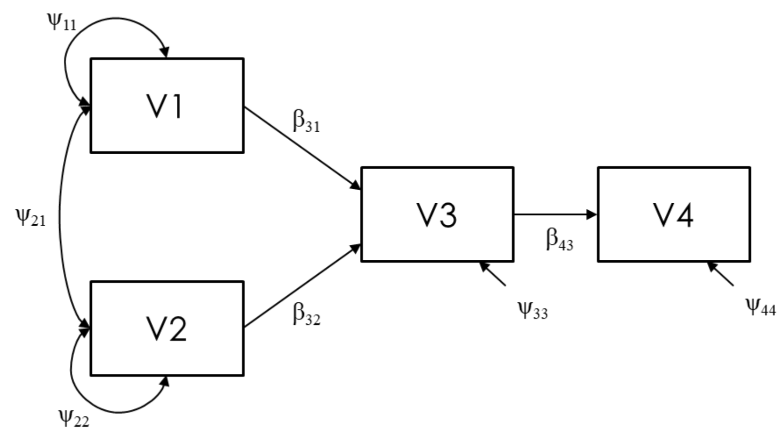
Example of a factor model



What is SEM?

The covariance matrix between the observed variables is modeled as a function of SEM parameters

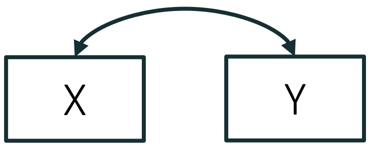
	V1	V2	V3	V4
V1	σ_{11}			
V2	σ_{21}	σ_{22}		
V3	σ_{31}	σ_{32}	σ_{33}	
V4	σ_{41}	σ_{42}	σ_{43}	σ_{44}



$$\left|\begin{array}{llll}
\psi_{11} & & & \\
\psi_{21} & \psi_{22} & & \\
\beta_{31}\psi_{11}+\beta_{32}\psi_{21} & \beta_{31}\psi_{21}+\beta_{32}\psi_{22} & (\beta_{31}\psi_{11}+\beta_{32}\psi_{21})\beta_{31}+ & \\
 & & (\beta_{31}\psi_{21}+\beta_{32}\psi_{22})\beta_{32}+\psi_{33} & \\
\beta_{31}\beta_{43}\psi_{11}+ & \beta_{31}\beta_{43}\psi_{21}+ & (\beta_{31}\beta_{43}\psi_{11}+\beta_{32}\beta_{43}\psi_{21})\beta_{31}+ & (\beta_{31}\beta_{43}\psi_{11}+\beta_{32}\beta_{43}\psi_{21})\beta_{31}\beta_{43}+ \\
\beta_{32}\beta_{43}\psi_{21} & \beta_{32}\beta_{43}\psi_{22} & (\beta_{31}\beta_{43}\psi_{21}+\beta_{32}\beta_{43}\psi_{22})\beta_{32}+ & (\beta_{31}\beta_{43}\psi_{21}+\beta_{32}\beta_{43}\psi_{22})\beta_{32}\beta_{43}+ \\
 & & \beta_{43}\psi_{33} & \beta_{43}^{2}\psi_{33}+\psi_{44}
\end{array}\right|$$

Meta-analytic structural equation modeling

Combining meta-analyse (MA) and structural equation modeling (SEM)



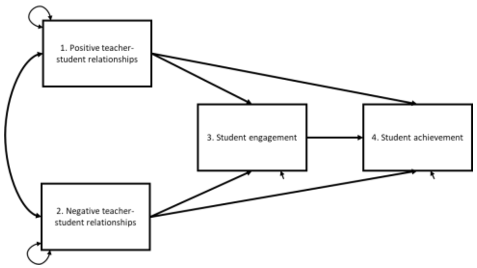
Standard meta-analysis

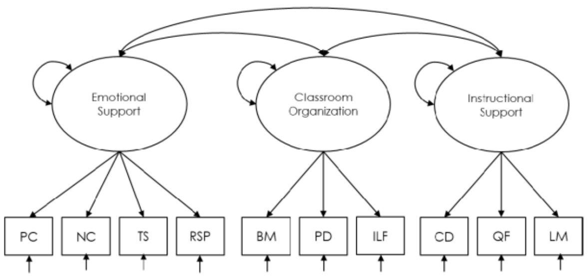
Complete theoretical models

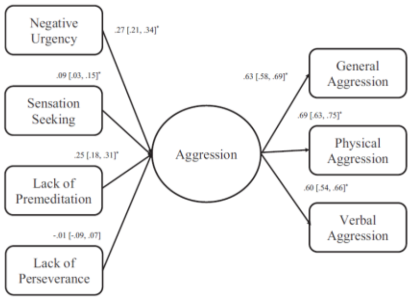
Mediating variables

Model fit

Latent variables

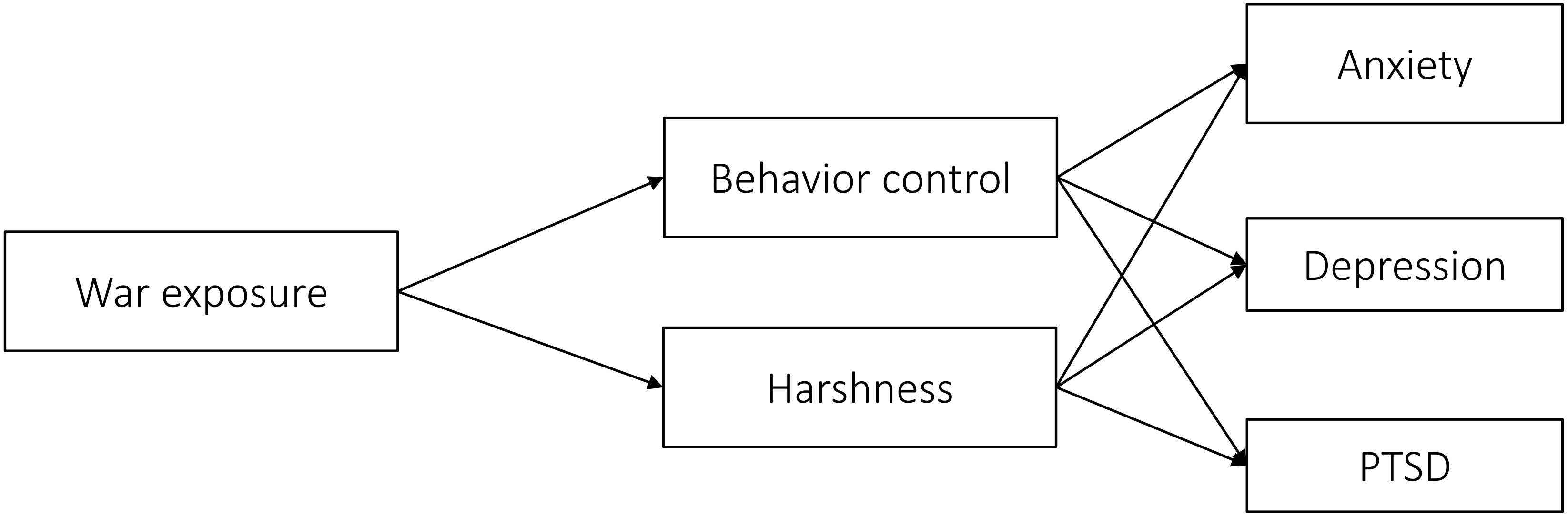






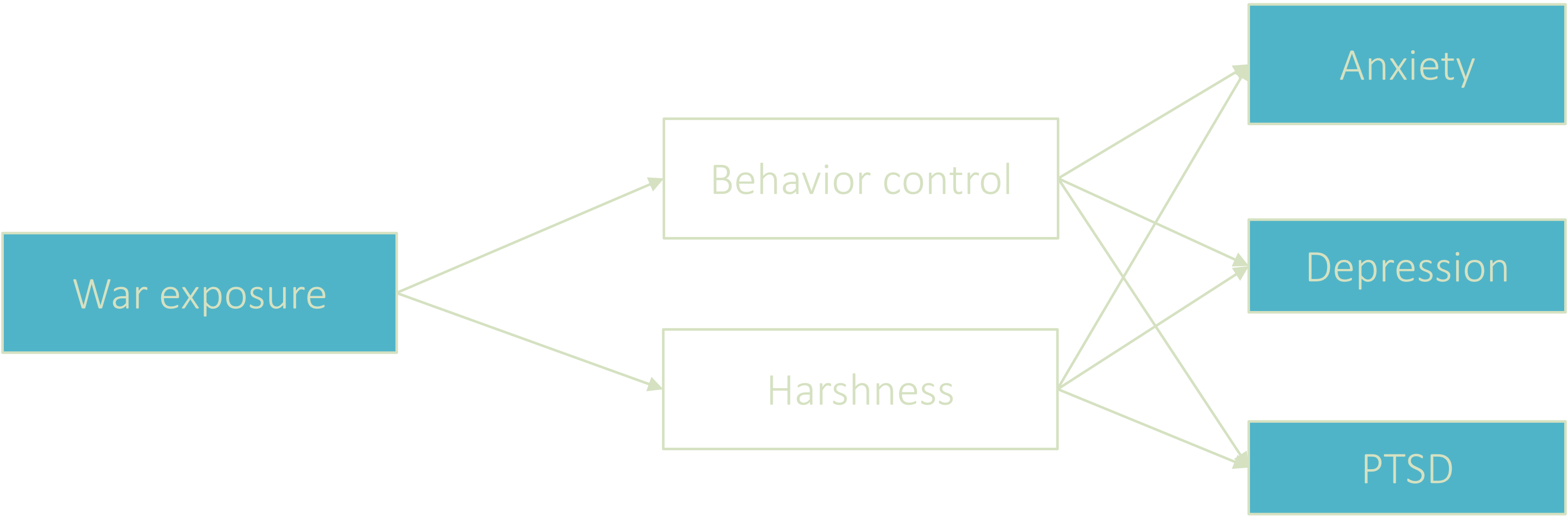
Example

Eltanamly et al. (2021)



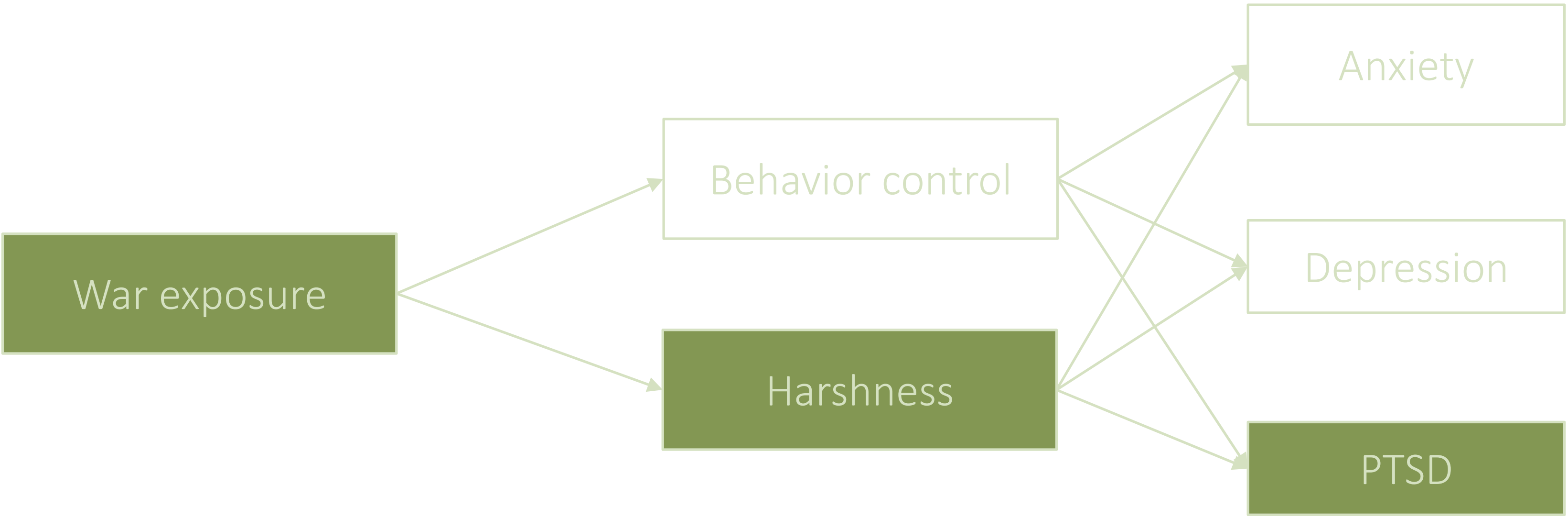
War → Parenting behavior→ Child behavioral problems

Study 1

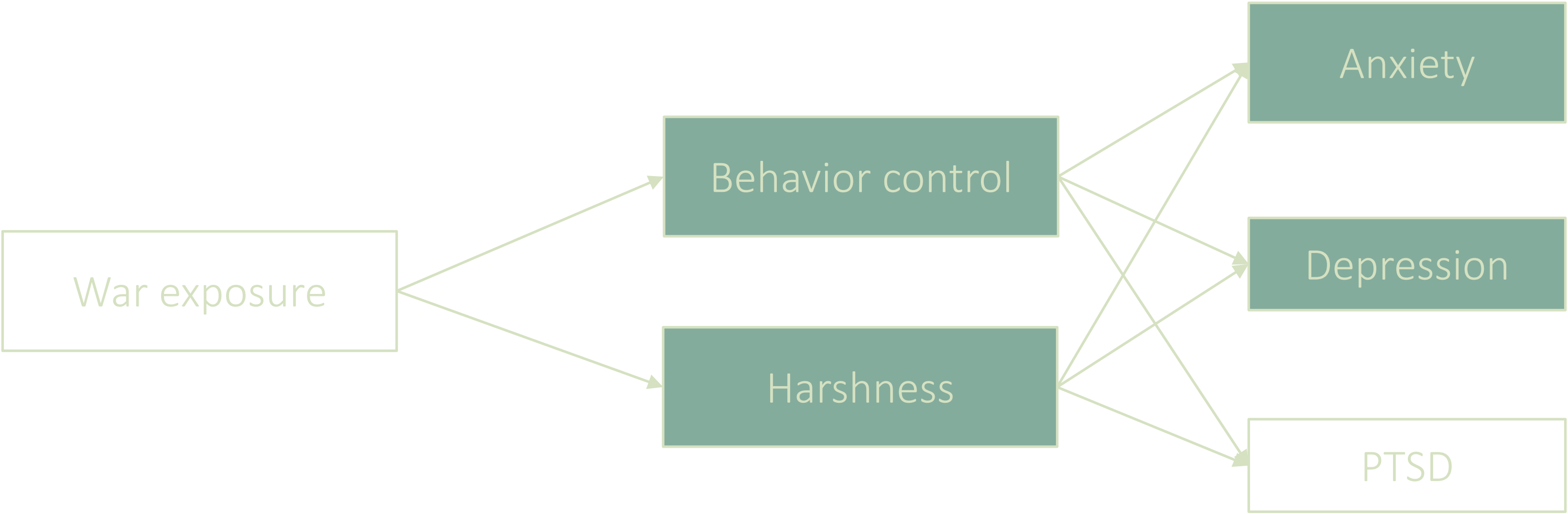


War exposure
Behavior control
Harshness
Anxiety
Depression
PTSD

Study 2

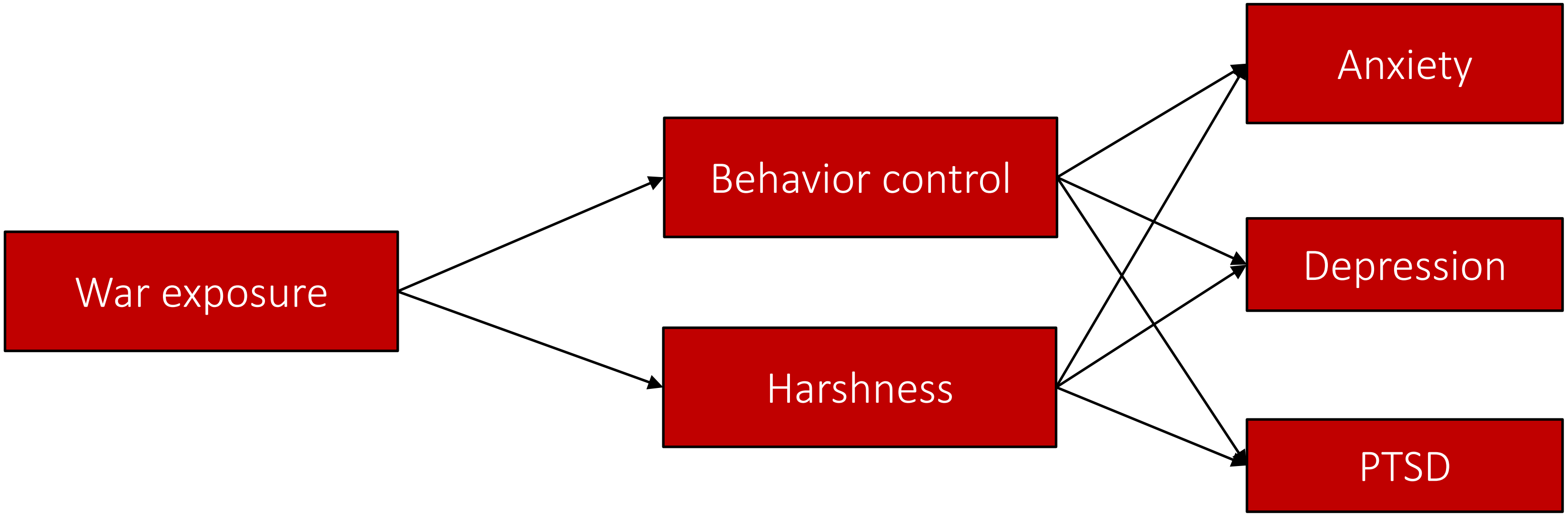

War exposure
Behavior control
Harshness
Anxiety
Depression
PTSD

Study k



War exposure
Behavior control
Harshness
Anxiety
Depression
PTSD

Eltanamly et al. (2021)
38 studies



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Factors affecting adoption of online banking: A meta-analytic structural equation modeling study

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The Structure of Common Emotion Regulation Strategies:
A Meta-Analytic Examination

Kristin Naragon-Gainey, Tierney P. McMahon, and Thomas P. Chacko
University at Buffalo, the State University of New York

Review Article

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Neuropsychology Review (2020) 30:51–96
https://doi.org/10.1007/s11065-019-09423-6

REVIEW

The Factor Structure of Cognitive Functioning in Cognitively Healthy Participants: a Meta-Analysis and Meta-Analysis of Individual Participant Data

Joost A. Agelink van Rentergem[1,2] • Nathalie R. de Vent[1] • Ben A. Schmand[1,3,4] • Jaap M. J. Murre[1] • Janneke P. C. Staaks[5] • ANDI Consortium[5] • Hilde M. Huizenga[1,4,6]

...es

...of the big five personality traits
...A meta-analytic path-analysis

...cke[c], Dongseop Lee[d], Kayla Sergent[a]

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Methods for MASEM

Option 1: Pool correlations (or covariances), then fit SEM

- Univariate-r (Viswesvaran and Ones, 1995)
- GLS (Becker, 1992)
- Two Stage SEM (Cheung and Chan, 2005; Cheung, 2015)
- One-stage MASEM (Jak and Cheung, 2020)

Option 2: Fit SEM, then pool the SEM parameters

Becker and Wu (2007)

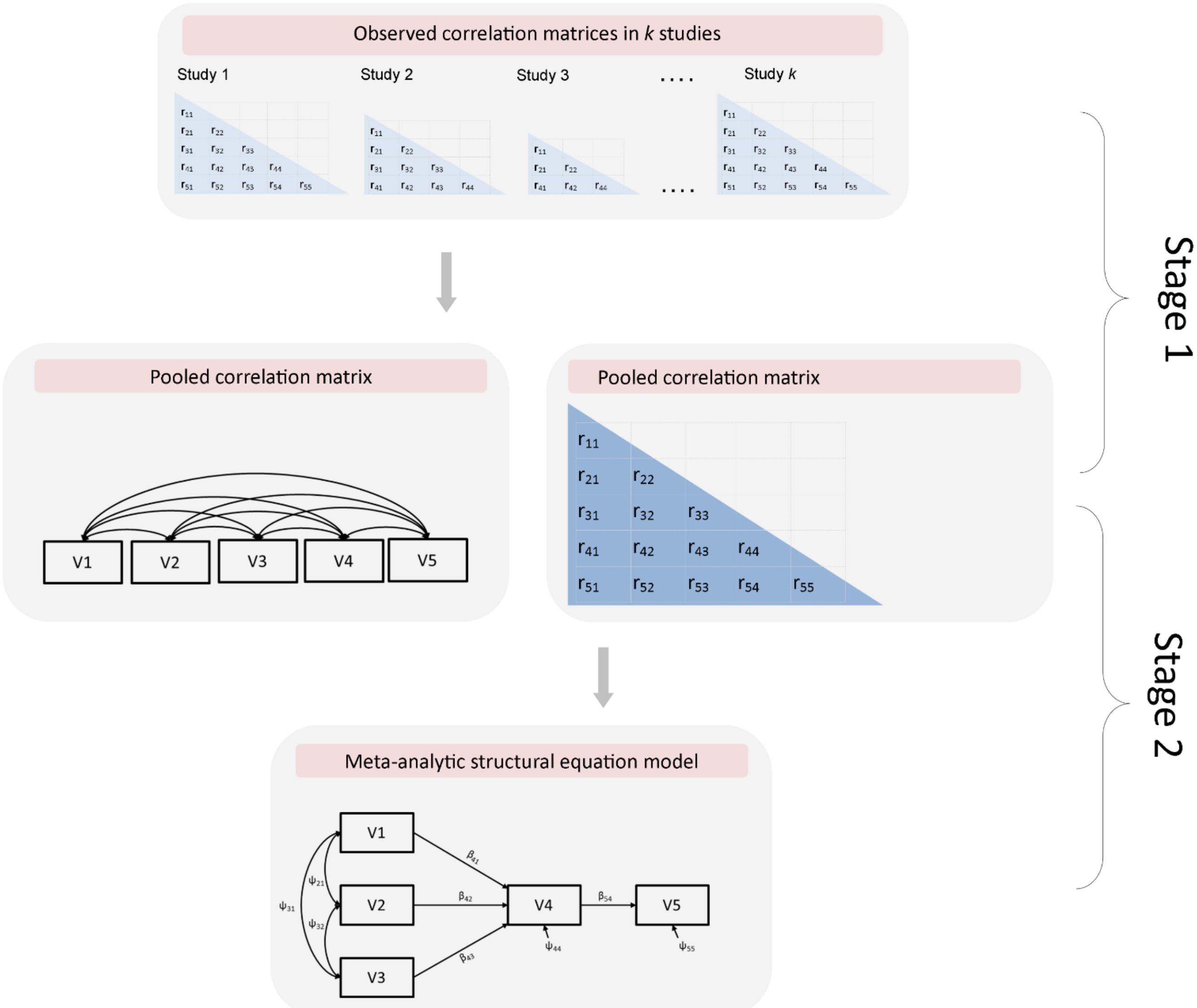
Gnambs and Staufenbiel (2016)

Disadvantage: Need complete data, and model should fit equally well in all samples

Ke, Zhang and Tong (2019)

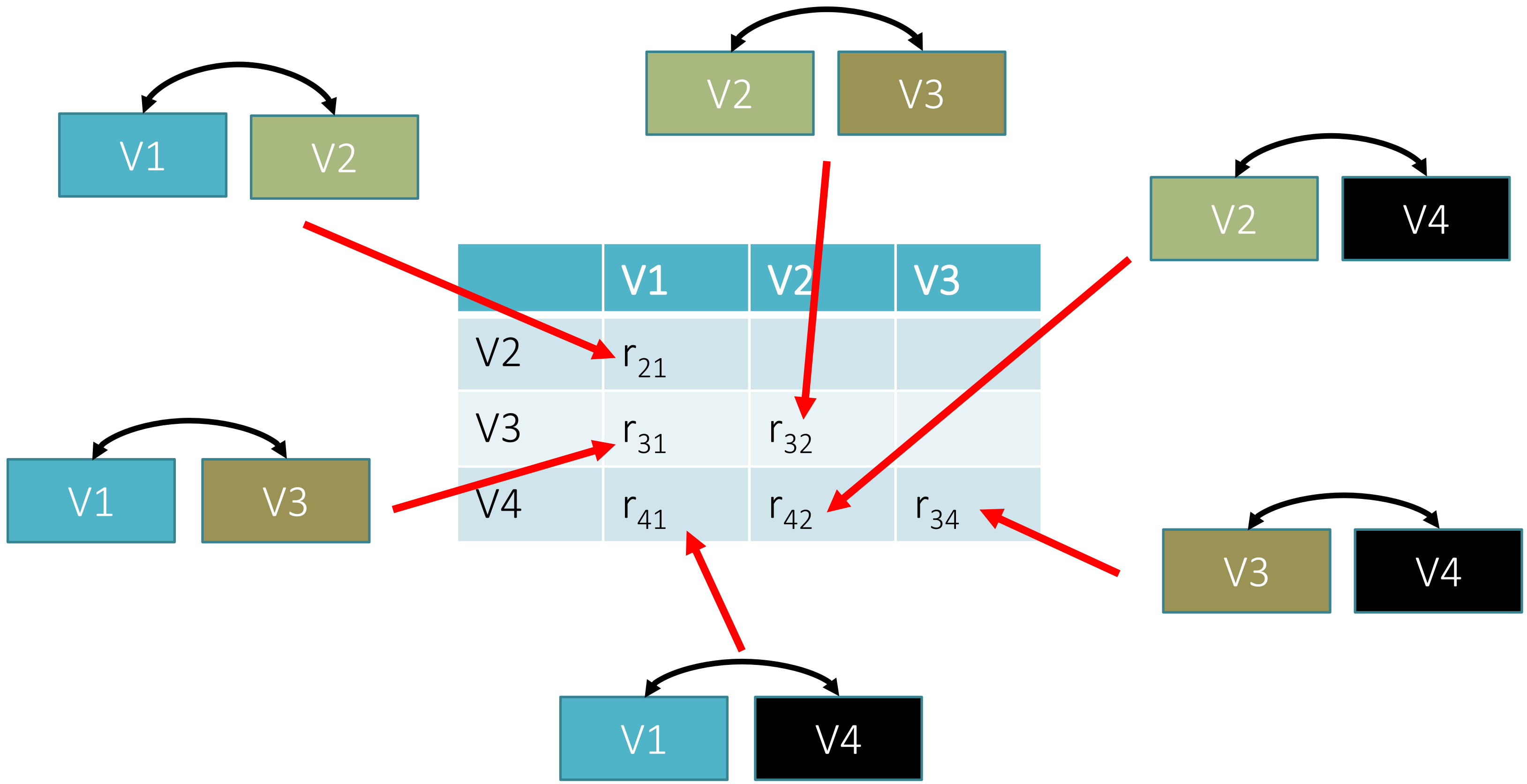
Bayesian method, solves the issues with parameter-based MASEM

Methods for MASEM



Univariate-r approach

Stage 1: Use several univariate meta-analyses to pool each correlation coefficient

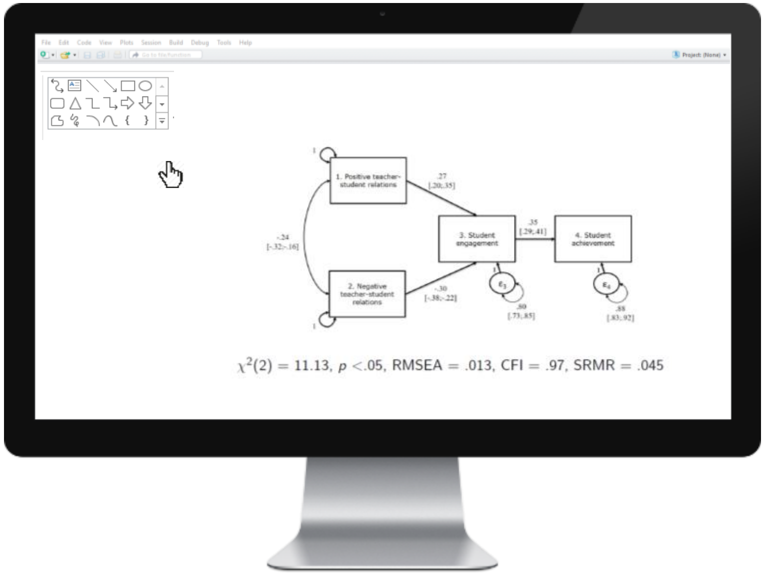


	V1	V2	V3
V2	r_{21}		
V3	r_{31}	r_{32}	
V4	r_{41}	r_{42}	r_{34}

Univariate-r approach

Stage 2: Fit the path model on the pooled correlation matrix

	V1	V2	V3
V2	r_{21}		
V3	r_{31}	r_{32}	
V4	r_{41}	r_{42}	r_{34}



Univariate-r approach

Stage 1: Use several univariate meta-analyses to pool each correlation coefficient

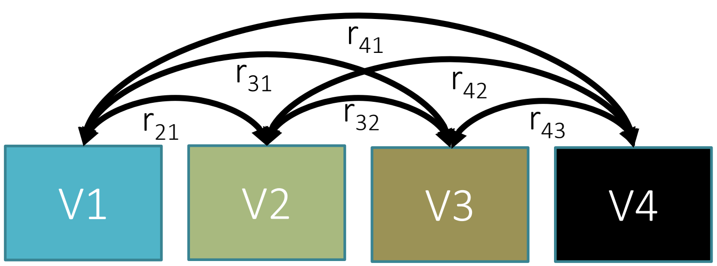
- Ignores sampling covariance
- Can lead to non-positive definite correlation matrices

Stage 2: Fit the path model on the pooled correlation matrix using standard SEM software

- What is the sample size?
- Differences in precision of estimated correlations ignored
- Between-studies variance not taken into account

Multivariate approaches

GLS, TSSEM and one-stage MASEM involve multivariate meta-analysis of correlation coefficients



	V1	V2	V3
V2	r_{21}		
V3	r_{31}	r_{32}	
V4	r_{41}	r_{42}	r_{34}

Estimates of average correlations: $\widehat{\boldsymbol{\rho}}_R$
And between-study (co)variances of correlations across studies: $\widehat{T^2}$

Between-studies heterogeneity

GLS, TSSEM and one-stage MASEM

4 variables
6 mean correlations:

	V1	V2	V3
V2	r_{21}		
V3	r_{31}	r_{32}	
V4	r_{41}	r_{42}	r_{43}

Between-studies heterogeneity

GLS, TSSEM and one-stage MASEM

With 6 mean correlations:

6 variances
15 covariances

In practice:
Use diagonal $\mathbf{T}^2$
(Becker & Aloe, 2019)

Between-studies covariance matrix $\mathbf{T}^2$

	r_{21}	r_{31}	r_{41}	r_{32}	r_{42}	r_{43}
r_{21}	τ^2_{11}					
r_{31}	τ_{21}	τ^2_{22}				
r_{41}	τ_{31}	τ_{32}	τ^2_{33}			
r_{32}	τ_{41}	τ_{42}	τ_{43}	τ^2_{44}		
r_{42}	τ_{51}	τ_{52}	τ_{53}	τ_{54}	τ^2_{55}	
r_{43}	τ_{61}	τ_{62}	τ_{63}	τ_{64}	τ_{65}	τ^2_{66}

TSSEM

Stage 1: Random effects multivariate meta-analysis of correlation coefficients using ML estimation

$$\boldsymbol{r}_i = \boldsymbol{\rho}_R + \boldsymbol{u}_i + \boldsymbol{\varepsilon}_i$$

$\text{cov}(\boldsymbol{\varepsilon}_i) = V_i$ (Within studies covariances)

$\text{cov}(\mathbf{u}_i) = T^2$ (Between studies (co)variances)

Stage 2: Fit SEM with WLS

$$\begin{vmatrix} \psi_{11} & & & \\ \psi_{21} & \psi_{22} & & \\ \beta_{31}\psi_{11} + \beta_{32}\psi_{21} & \beta_{31}\psi_{21} + \beta_{32}\psi_{22} & \begin{array}{l}(\beta_{31}\psi_{11} + \beta_{32}\psi_{21})\beta_{31} + \\ (\beta_{31}\psi_{21} + \beta_{32}\psi_{22})\beta_{32} + \psi_{33}\end{array} & \\ \begin{array}{l}\beta_{31}\beta_{43}\psi_{11} + \\ \beta_{32}\beta_{43}\psi_{21}\end{array} & \begin{array}{l}\beta_{31}\beta_{43}\psi_{21} + \\ \beta_{32}\beta_{43}\psi_{22}\end{array} & \begin{array}{l}(\beta_{31}\beta_{43}\psi_{11} + \beta_{32}\beta_{43}\psi_{21})\beta_{31} + \\ (\beta_{31}\beta_{43}\psi_{21} + \beta_{32}\beta_{43}\psi_{22})\beta_{32} + \\ \beta_{43}\psi_{33}\end{array} & \begin{array}{l}(\beta_{31}\beta_{43}\psi_{11} + \beta_{32}\beta_{43}\psi_{21})\beta_{31}\beta_{43} + \\ (\beta_{31}\beta_{43}\psi_{21} + \beta_{32}\beta_{43}\psi_{22})\beta_{32}\beta_{43} + \\ \beta_{43}^2\psi_{33} + \psi_{44}\end{array} \end{vmatrix}$$

$$\widehat{\boldsymbol{F}}_{\text{WLS}} = (\widehat{\boldsymbol{\rho}}_R - \widehat{\boldsymbol{\rho}}_{MODEL})^{\top}\, \widehat{\boldsymbol{V}}_R^{\,-1}\, (\widehat{\boldsymbol{\rho}}_R - \widehat{\boldsymbol{\rho}}_{MODEL})$$

	V1	V2	V3	V4
V1	σ_{11}			
V2	σ_{21}	σ_{22}		
V3	σ_{31}	σ_{32}	σ_{33}	
V4	σ_{41}	σ_{42}	σ_{43}	σ_{44}

GLS approach

Stage 1: Random effects multivariate meta-analysis of correlation coefficients

$$\boldsymbol{r} = \boldsymbol{\rho}_R + \boldsymbol{u} + \boldsymbol{\varepsilon}$$

$\widehat{\boldsymbol{\rho}}_R$ = $(X^T \Sigma^{-1} X)^{-1} X^T \Sigma^{-1} r$

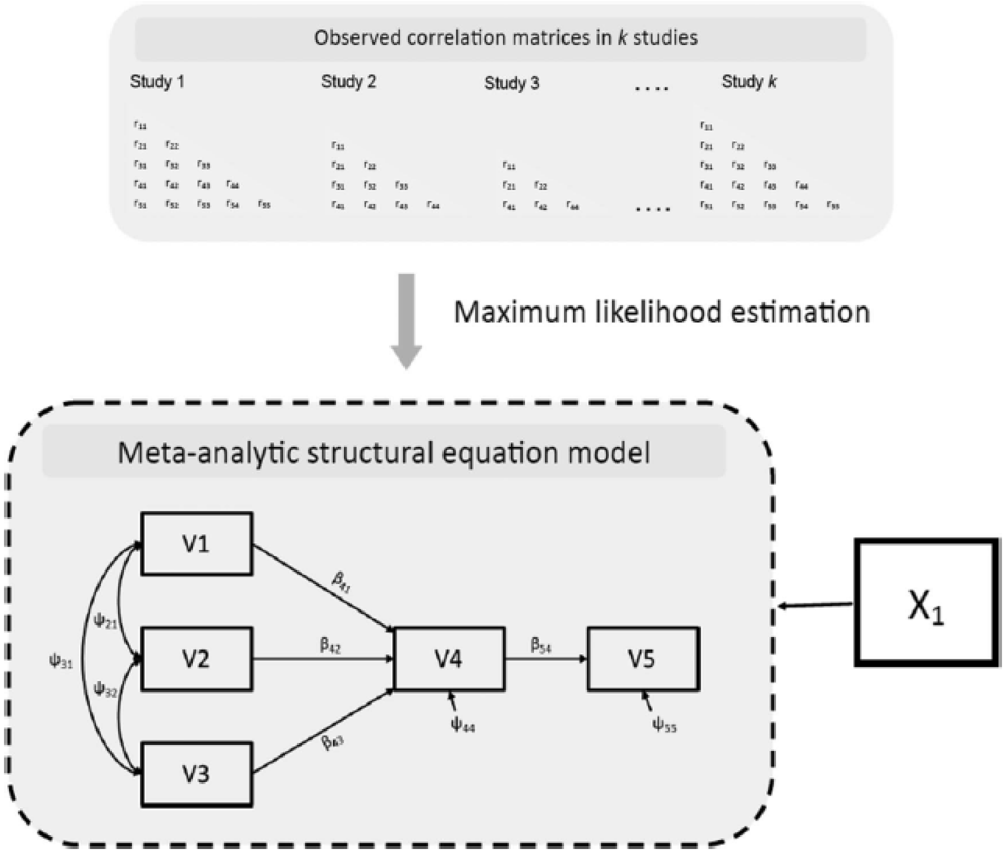
$\widehat{\boldsymbol{V}}_R$ = $(X^T \Sigma^{-1} X)^{-1}$

r = vector of observed correlations
X = stacked selection matrices
Σ = block-diagonal matrix with $V_i + T^2$

Stage 2: $\widehat{\boldsymbol{B}} = \widehat{\boldsymbol{\rho}}_{XX}{}^{-1}\, \widehat{\boldsymbol{\rho}}_{YX}$

Alternative: use WLS with $\widehat{\boldsymbol{\rho}}_R$ and $\widehat{\boldsymbol{V}}_R$ like in TSSEM

One-stage MASEM



One-stage MASEM

Multivariate random-effects analysis of correlation coefficients

$$\boldsymbol{r}_i = \boldsymbol{\rho}_R + \boldsymbol{u}_i + \boldsymbol{\varepsilon}_i$$

One-stage MASEM restricts the pooled correlation matrix to a SEM model:

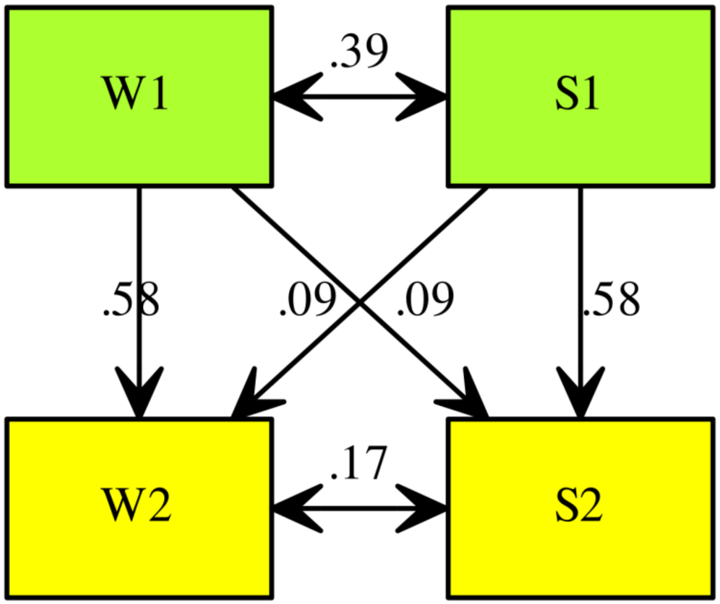
$$\boldsymbol{\rho}_R = \text{vechs}(\boldsymbol{\Lambda}\boldsymbol{\Phi}\boldsymbol{\Lambda}^{-1t} + \boldsymbol{\Theta})$$

The SEM parameters may be regressed on study-level moderator variables

Comparison of four methods

Data generating model

Based on Nohe et al. (2015)



Fit correct model using univariate-r, GLS, TSSEM and one-stage MASEM

Comparison of four methods

Fit correct path model using

Univariate-r

- Univariate meta-analyses of raw correlations, weighted by N
- Using harmonic mean as sample size in Stage 2

Multivariate methods (GLS, TSSEM, one-stage MASEM)

- With diagonal $\mathbf{T}^2$ (although generated full $\mathbf{T}^2$)

Conditions

Varying:

- Number of studies

  k = 16, 32 or 64

- Missing data

  0, 50 or 75% of the studies missed 2 of the 4 variables

- Heterogeneity (T^2)

  Between-studies SD .05 or .10
  Between studies correlations .10 or .30

Fixed:

- Sample size per study

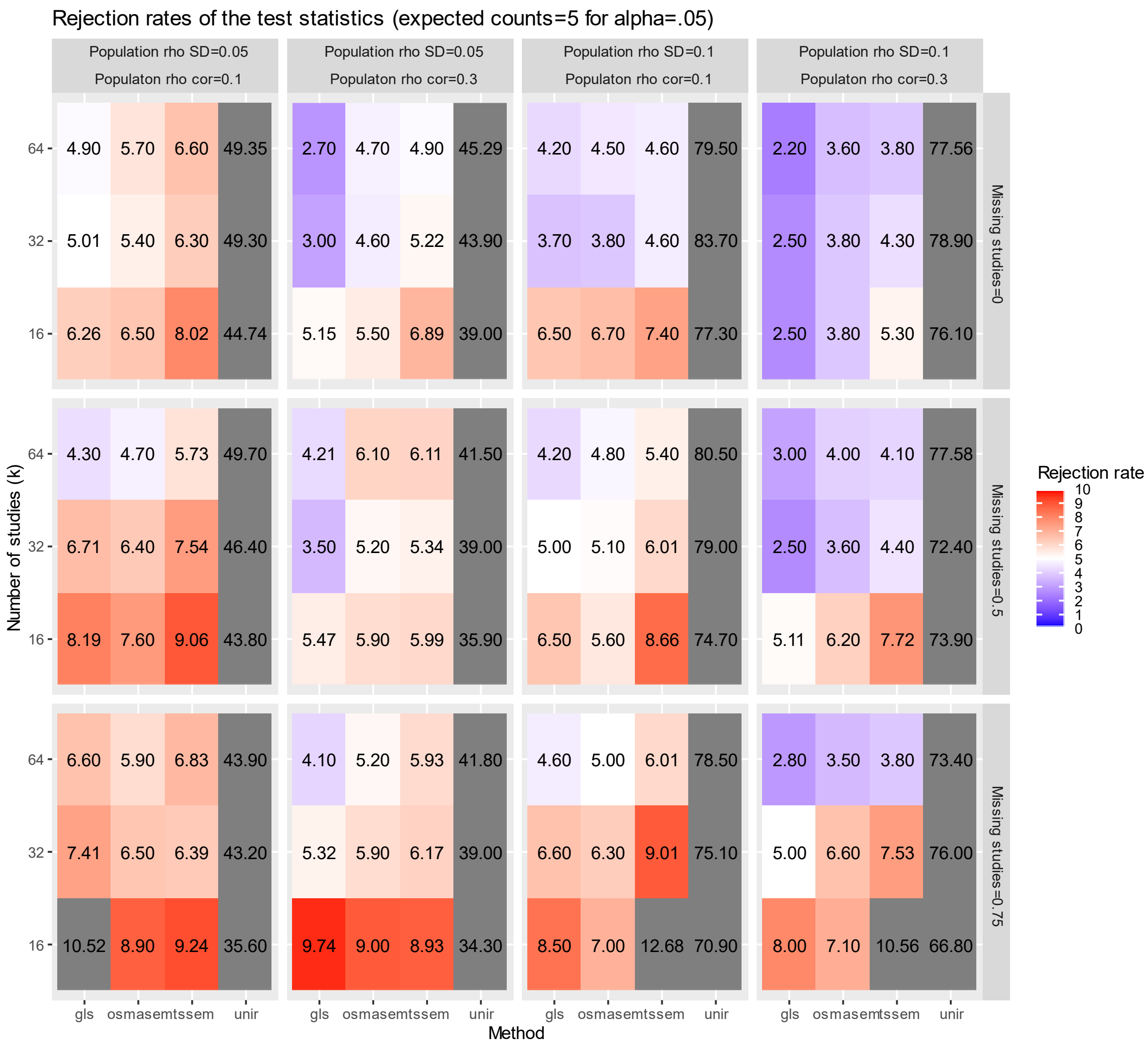
Evaluation

Rejection percentages of test statistic

Relative bias in parameter estimates

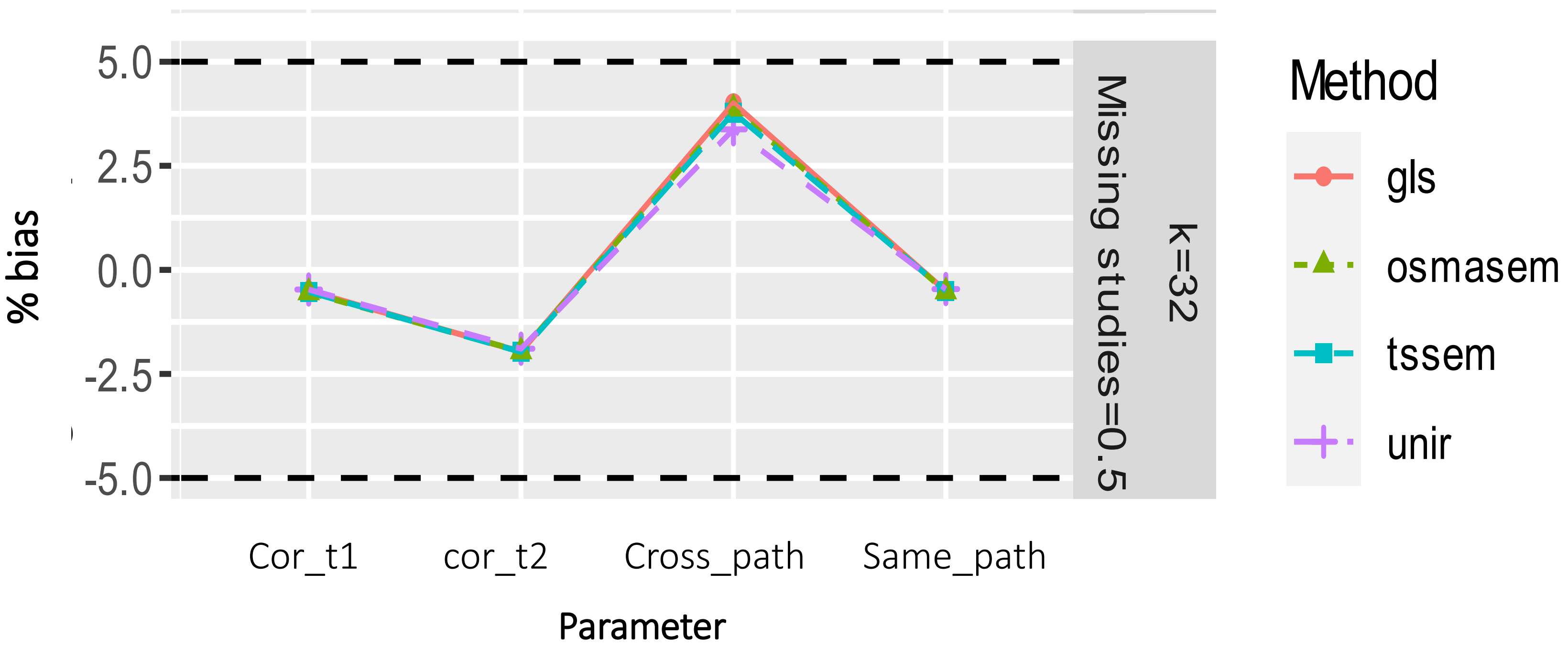
Relative bias in standard errors

Test statistic (rejection percentages)

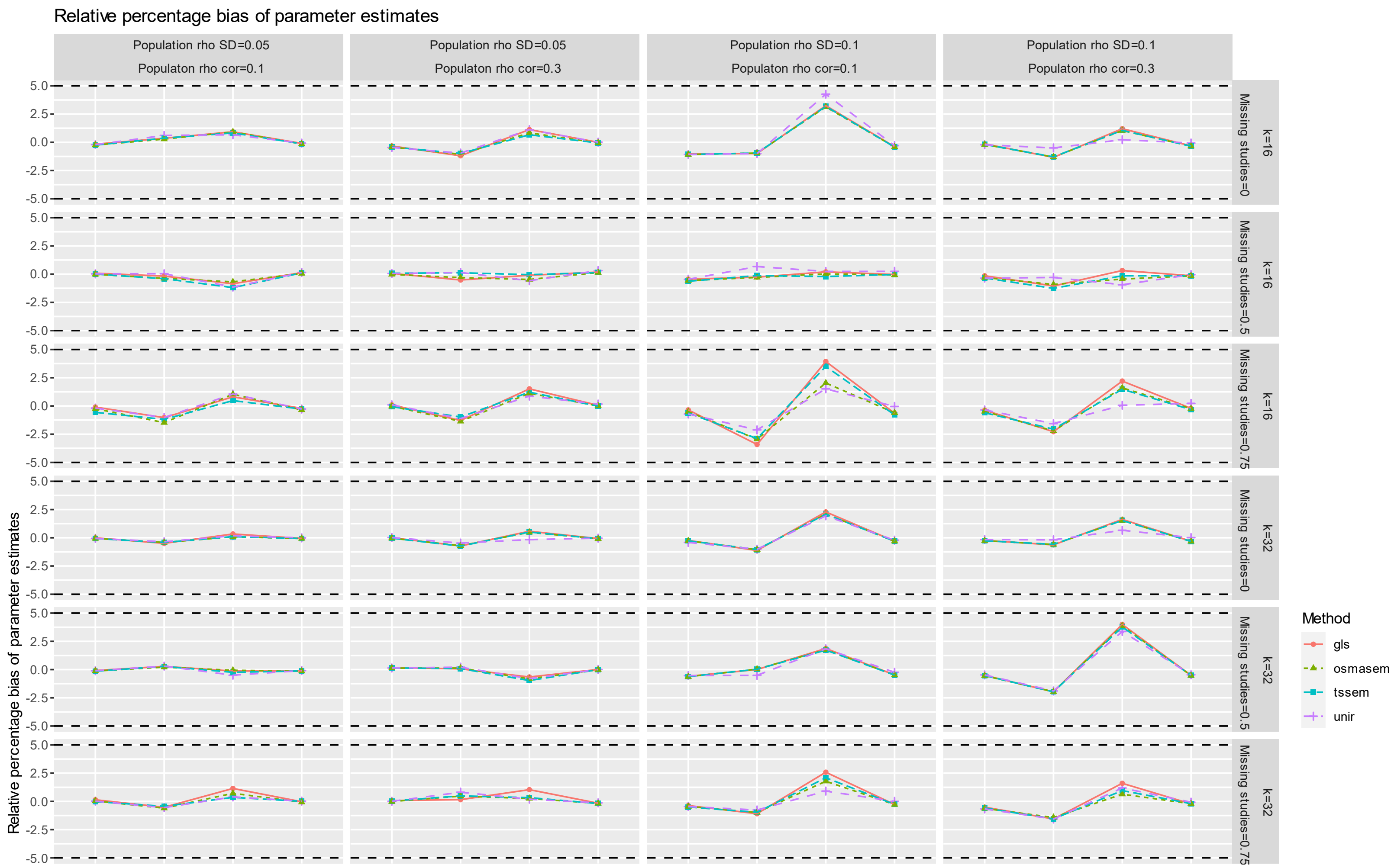


Bias in parameter estimates

K = 32, SD = 0.1, cor = 0.3, missing studies = 0.5

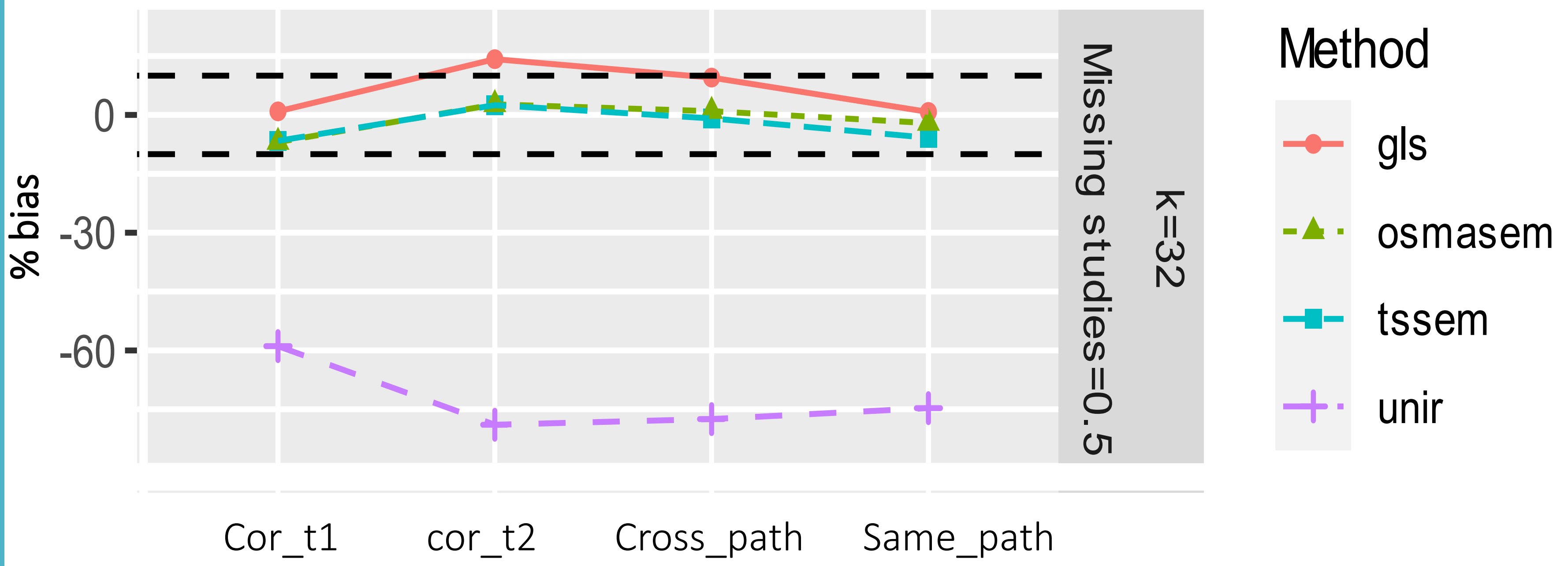


Bias in parameter estimates

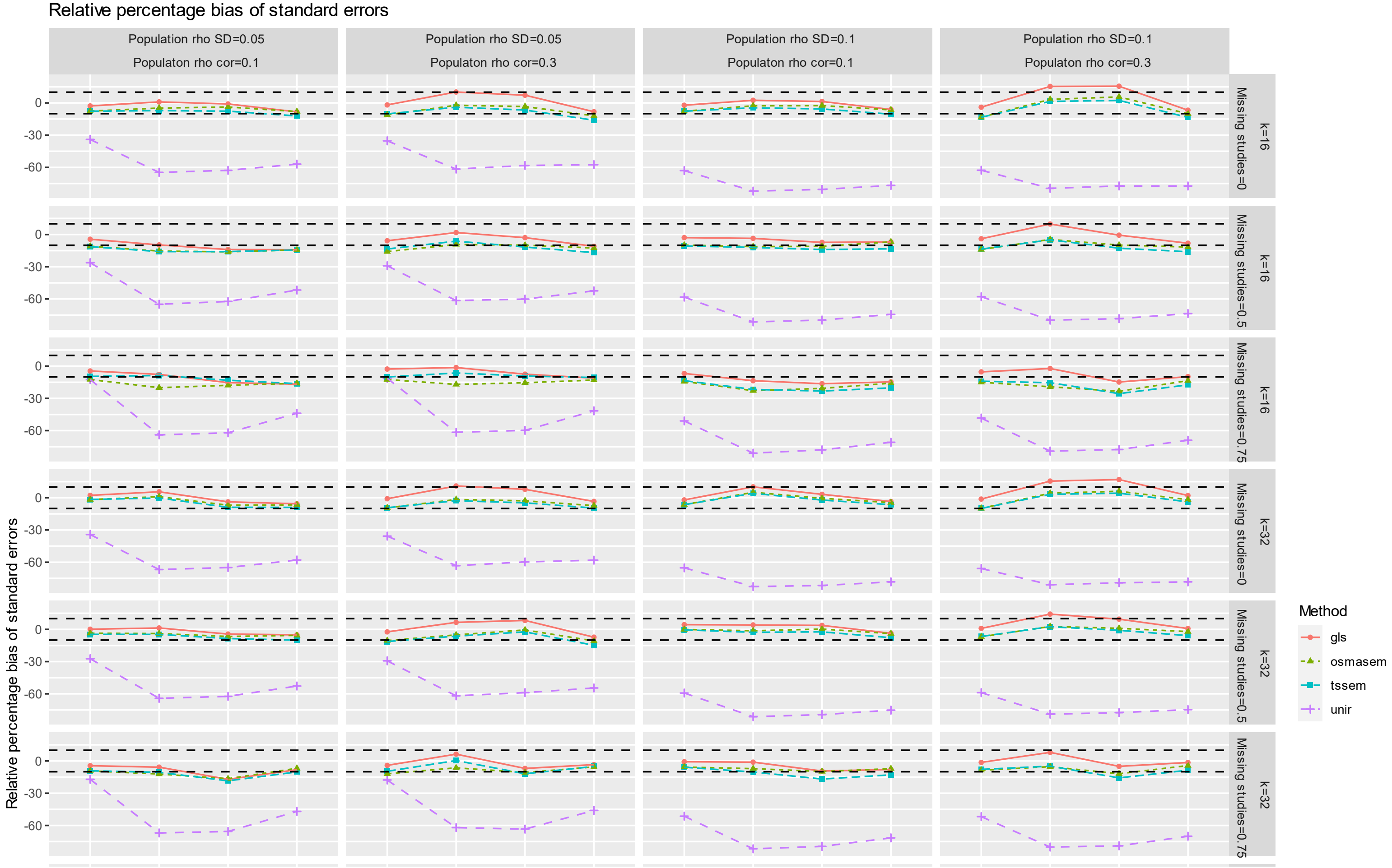


Bias in standard errors

K = 32, SD = 0.1, cor = 0.3, missing studies = 0.5



Bias in standard errors



Conclusions

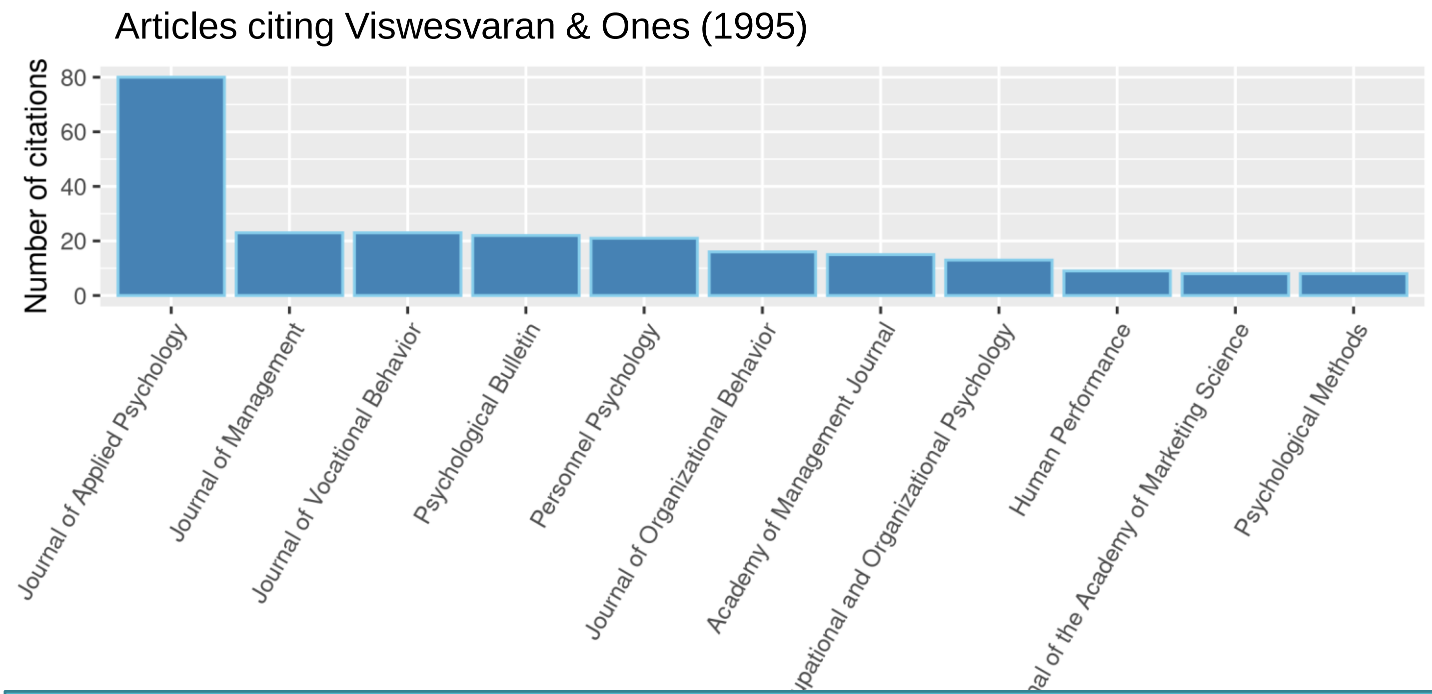
Univariate-r approach leads to unbiased parameter estimates, but:

<u>Extremely inflated </u>test-statistics and associated Type 1 errors

<u>Extremely biased </u>standard errors

The three multivariate methods generally lead to unbiased parameter estimates, well behaved test-statistics, and correct standard errors

Discussion



Within the Journal of Applied Psychology, all-but-one of the MASEM applications since 2020 used the univariate-r method

Discussion

Multivariate methods also need more research

- Minimum sample size conditions (e.g., how many studies are needed in which conditions?)
- Handling dependent effect sizes

However, it is worrying how often researchers apply the only method that clearly leads to wrong results in all situations

Discussion

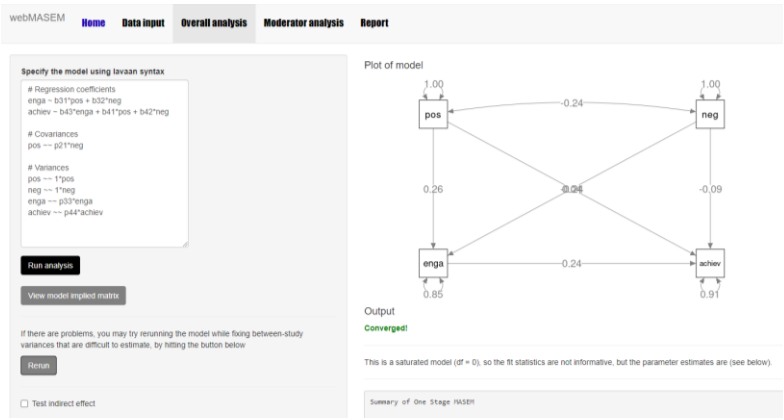
Why is the univariate method (still) so popular?

Because it is easier to apply?

Not anymore!

Tutorial and Shiny app for one-stage MASEM (Jak et al. 2021)

https://sjak.shinyapps.io/webMASEM/



Discussion

Why is the univariate method (still) so popular?

Because researchers copy the current practice I their field

Harder to change, but we are hopeful.

Thanks for listening!

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