



UNIVERSITY
OF AMSTERDAM

Robust Bayesian Meta-Regression

Model-Averaged Moderation Analysis in the Presence of Publication Bias

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with Maximilian Maier, Eric-Jan Wagenmakers, and Tom D. Stanley



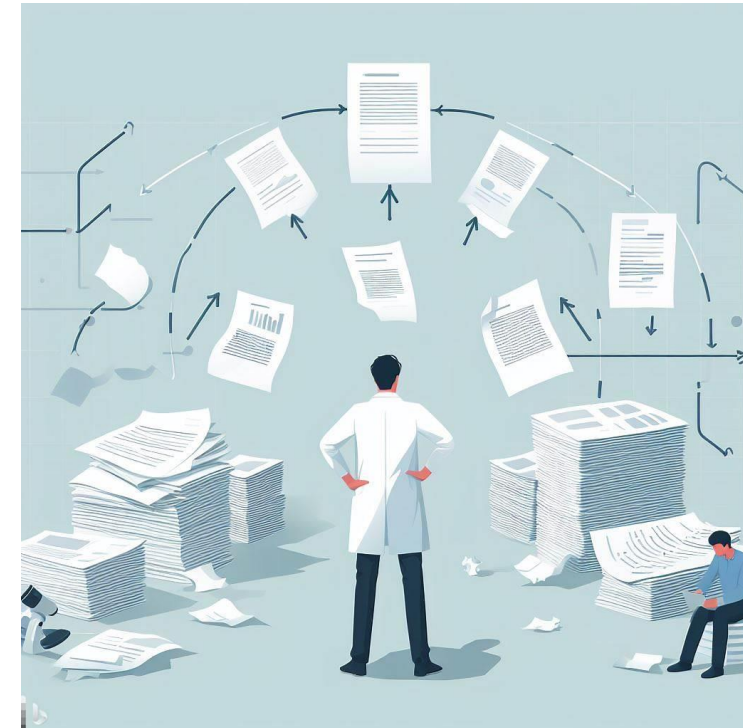
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Outline

- Meta-Analysis
- Publication bias
- Robust Bayesian meta-analysis
- Robust Bayesian meta-regression

Publication Bias

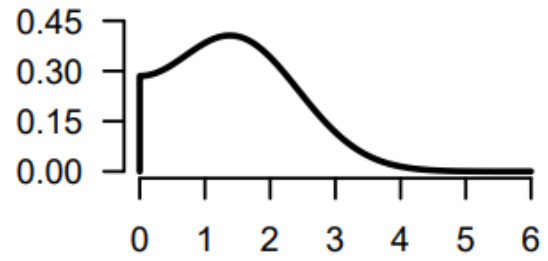
- Most large studies are likely to get published regardless of results
- Some moderately size studies might get lost if not convincing
- Many small studies won't be published unless statistically significant



Publication Bias

Power

0.3



Publication Bias

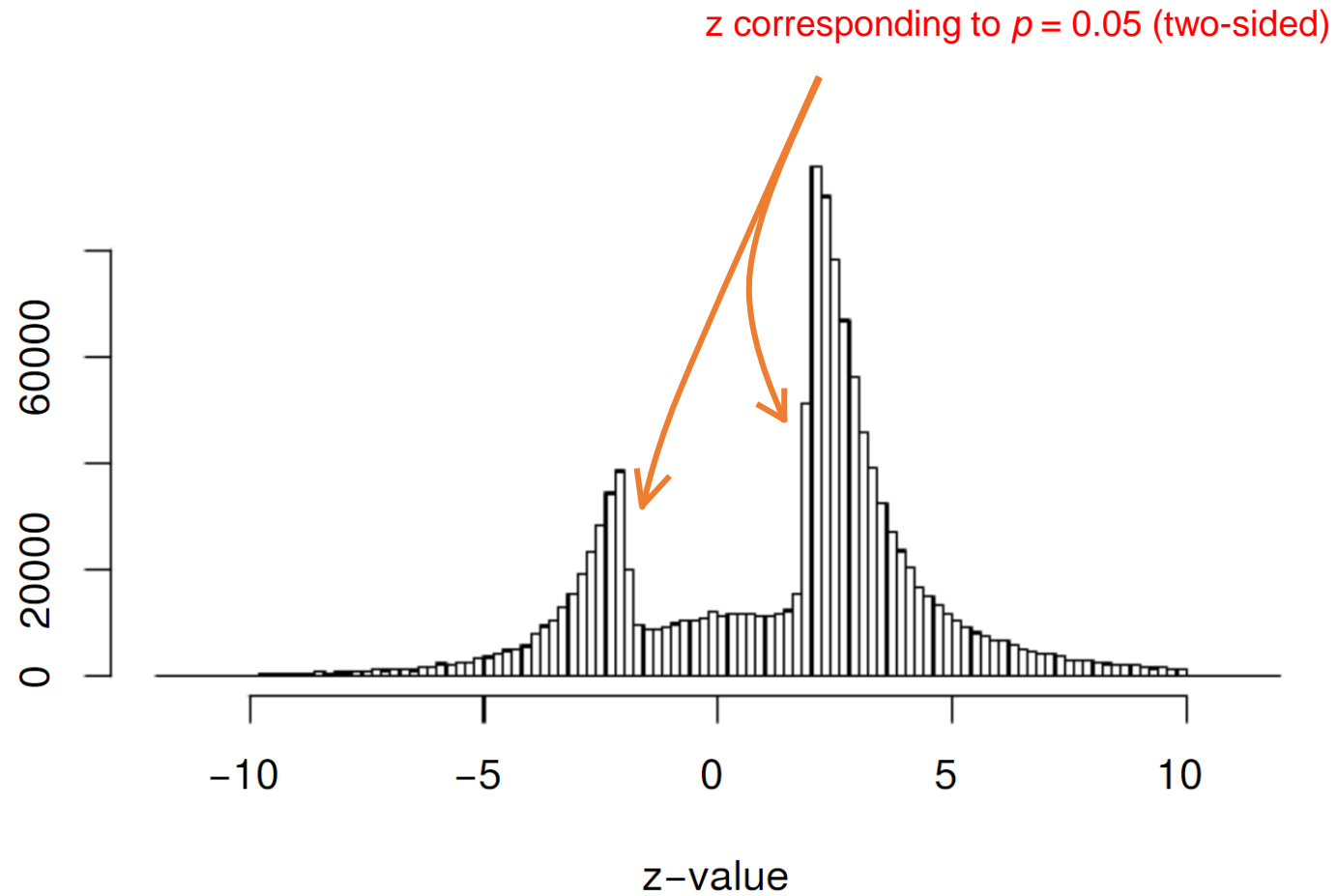


FIGURE 1 The distribution of more than one million z-values from Medline (1976–2019).

Meta-Analyses vs. RRR: Kvarven et al. (2020)

- Comparison of:
 - 15 meta-analyses from the field of psychology
 - Registered replication reports of a corresponding experiment
- The registered replication reports do not suffer from publication bias
=> should provide the best possible estimate of the true effect

Oppenheimer et al. (2009)

Tversky & Kahneman (1981)

Husnu & Crisp (2010)

Schwarz et al. (1991)

Hauser et al. (2007)

Critcher & Gilovich (2008)

Graham et al. (2009)

Jostmann et al. (2009)

Monin & Miller (2001)

Schooler
& Engstler-Schooler (1990)

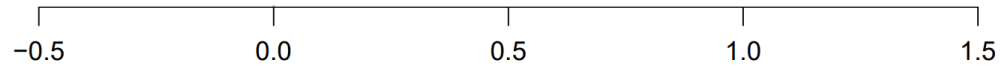
Sripada et al. (2014)

Rand et al. (2012)

Strack et al. (1988)

Srull & Wyer (1979)

Mazar et al. (2008)



■ RE ● RRR

Effect Size

Publication Bias Adjustment Methods

- Models adjusting for relationship between effect sizes and standard errors
 - Trim and fill (Duval & Tweedie, 2000)
 - PET-PEESE (Stanley & Doucouliagos, 2014)
 - EK (Bom & Rächinger, 2019)
- Selection models of p -values
 - 3PSM, 4PSM (Vevea & Hedges, 1995)
 - AK1, AK2 (Andrews & Kasy, 2019)
 - p -curve (Simonsohn et al., 2014)
 - p -uniform (Van Assen et al., 2015)

PET-PEESE

- Conditional meta-regression estimators
- Corrects for relationship between effect sizes and
 - Standard errors (PET)
 - Standard errors² (PEESE)
- Effect size estimate is based on
 - PET, if effect size test is not significant on $\alpha = 0.10$
 - PEESE, if effect size test is significant on $\alpha = 0.10$

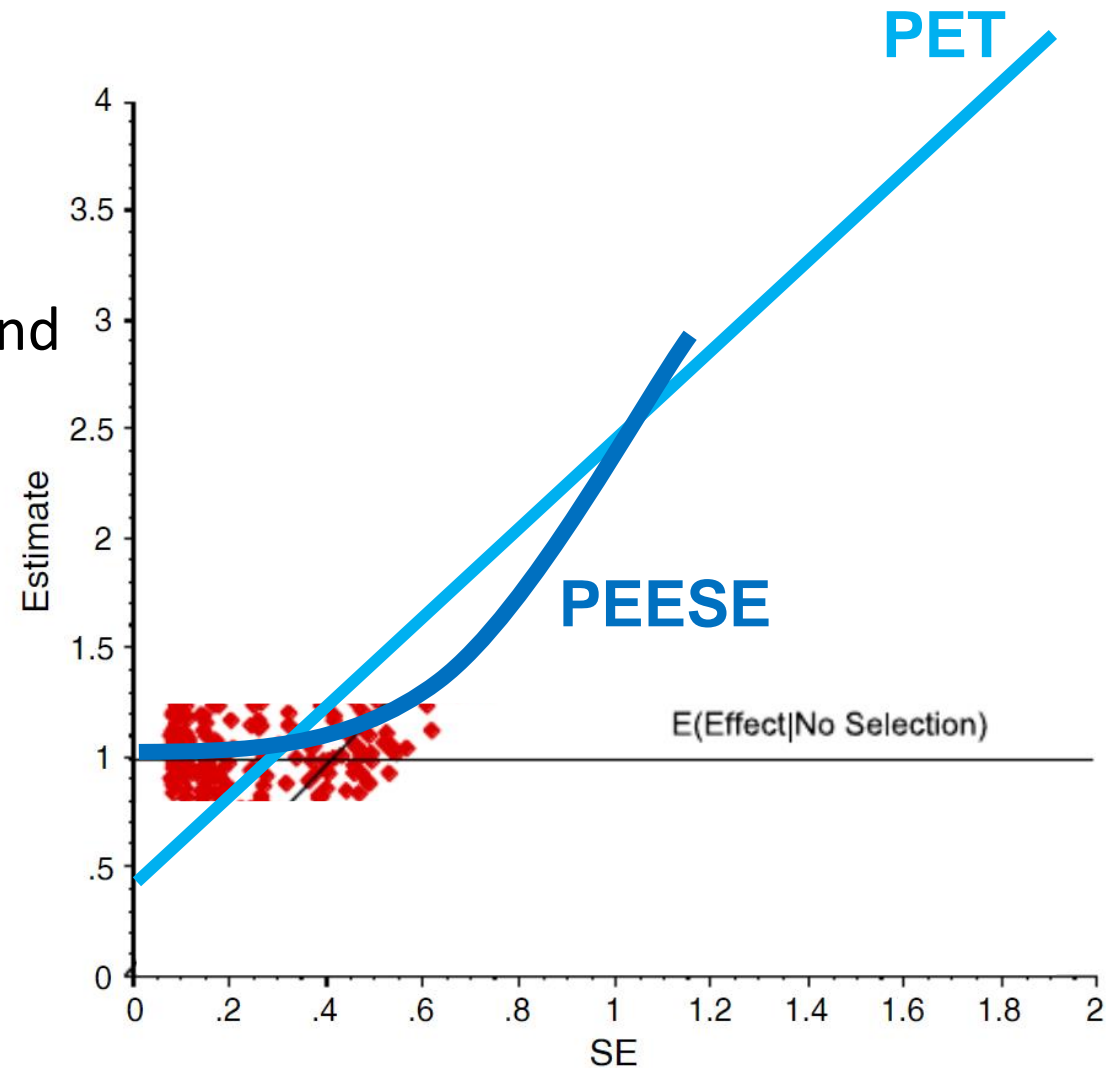
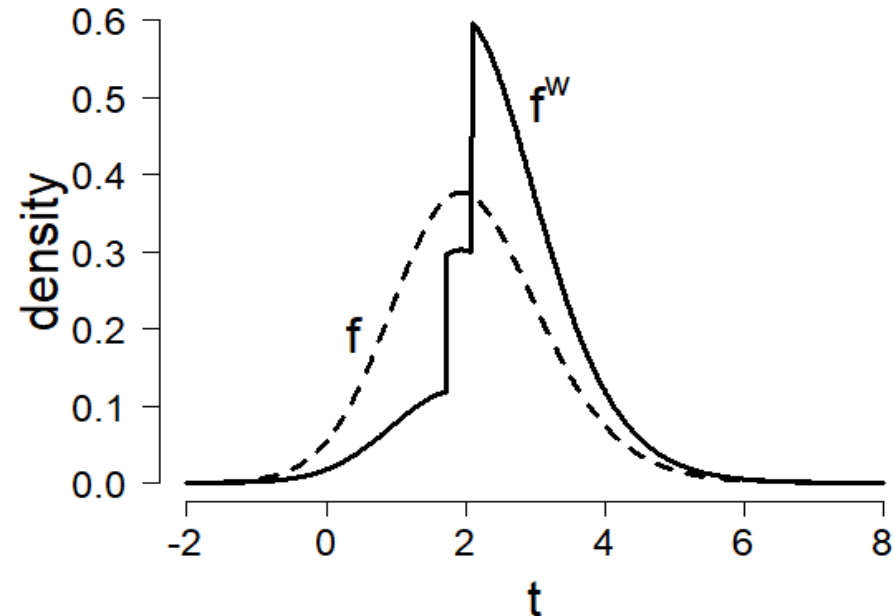
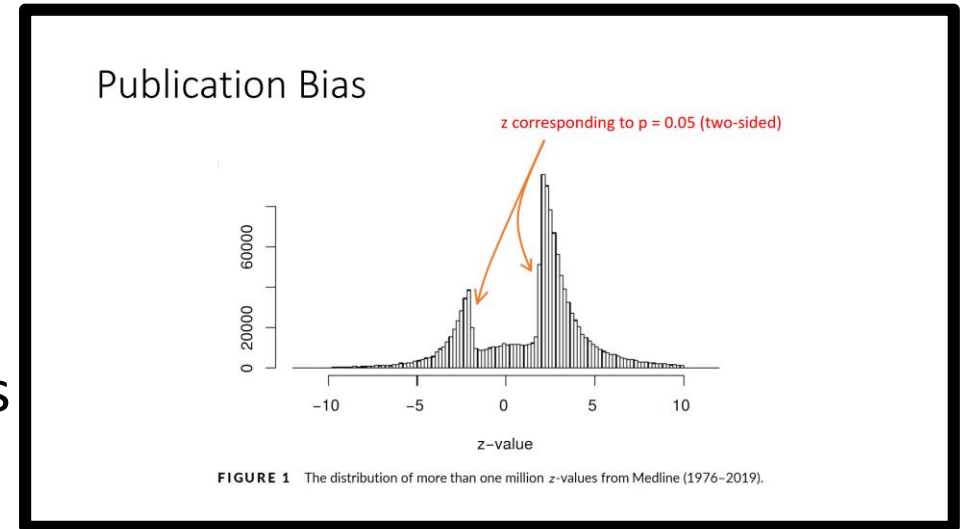
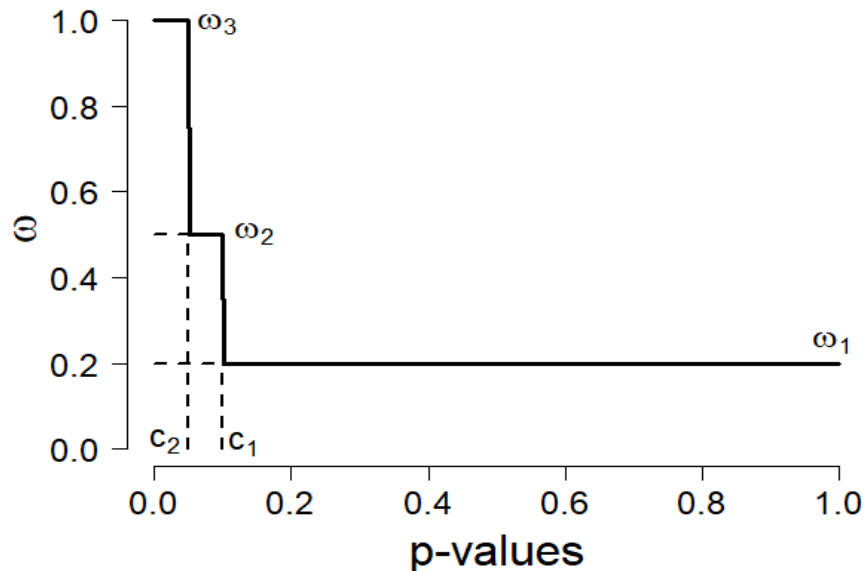
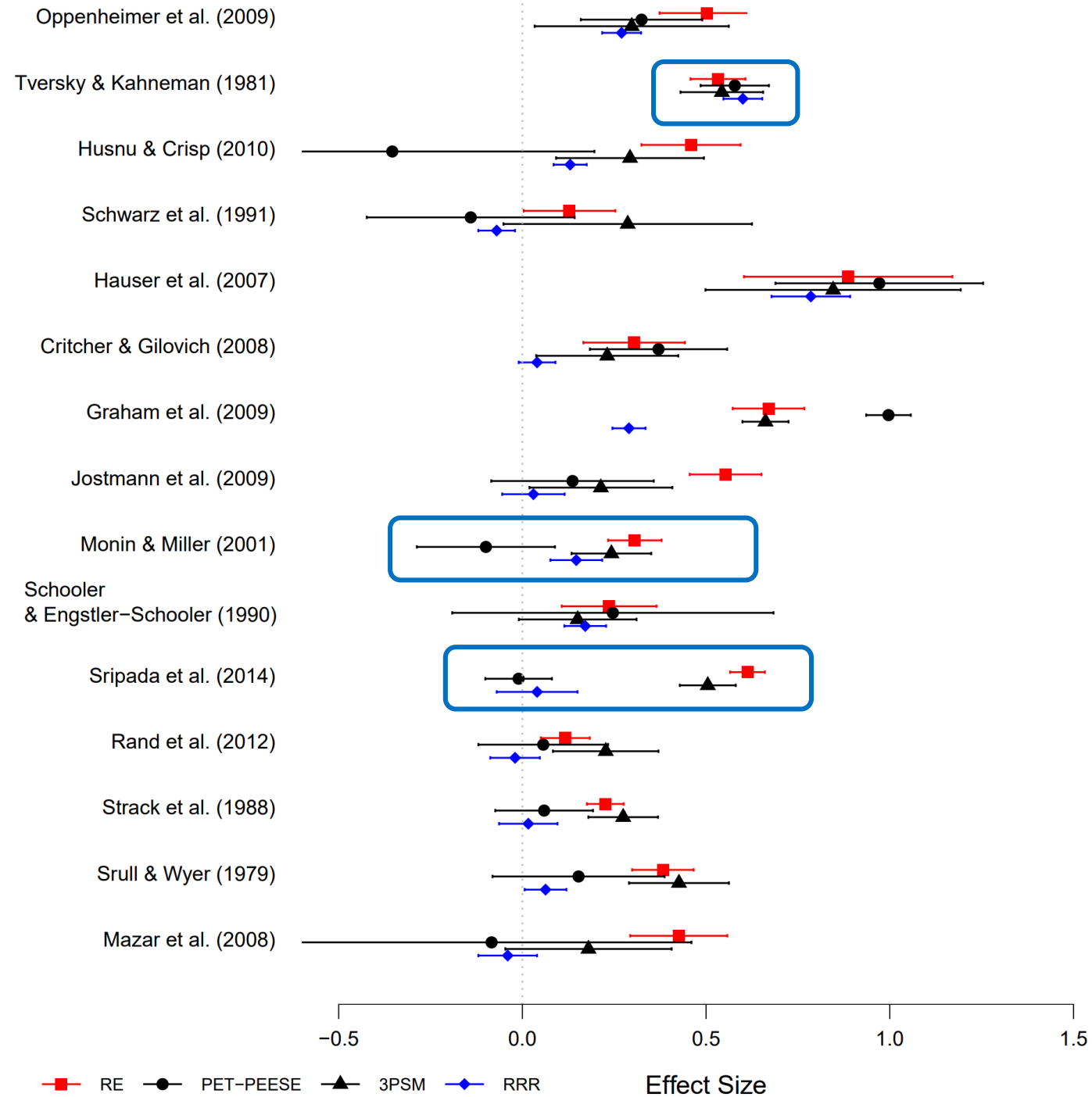


Figure 1. Plots 300 randomly generated yet selected effects (vertical axis) against their standard errors.

Selection Models

- Adjust for publication bias operating on p -values
- Meta-analytic models with:
 - Mean parameter μ
 - (Heterogeneity parameter τ)
 - Publication bias weights ω





Kvarven, A., Strömmland, E., & Johannesson, M. (2020). Comparing meta-analyses and preregistered multiple-laboratory replication projects. *Nature Human Behaviour*

Limitations of Existing Methods

- Require researchers to decide whether or not to adjust for publication bias in all-or-none fashion
- Cannot quantify evidence against publication bias; a non-significant p-value may indicate evidence of absence or absence of evidence
- Most fail under high between-study heterogeneity
- Poor performance in small samples and convergence issues

RoBMA – Robust Bayesian Meta-Analysis

- Bayesian model-averaging to base inference on multiple models simultaneously
(vs. deciding to adjust for publication bias in all-or-none fashion)
- Bayes factors to quantify evidence in favor of the presence or absence of effect/heterogeneity/publication bias
(vs. rejecting or failing to reject the null hypothesis)
- Prior distributions to regularize the estimates/incorporate prior knowledge
(vs. convergence problems/highly variable estimates under small sample sizes)
- Bayesian evidence updating independent of sampling plan
(vs. accumulation bias)

THE PRIOR MODEL DEMONS ALL SHOUT ...

“Only sampling variability!”
(Fixed-effect models)

“No bias here!”
(Unadjusted models)

“No effect here!”
(Null hypothesis models)

“Everything is biased!”
(Publication bias adjusted models)

“The treatment works!”
(Alternative hypothesis models)

“There is additional heterogeneity!”
(Random-effects models)

... BUT AFTER THEY GORGE ON DATA ...



... SOME POSTERIOR MODEL DEMONS BECOME POWERFUL, AND OTHERS WITHER AWAY ...

"No bias here!"
(Unadjusted models)

LEE!

"Only sampling variability!"
(Fixed-effect models)

"No effect here!"
(Null hypothesis models)

"Everything is biased!"
(Publication bias adjusted models)

"There is additional
heterogeneity!"
(Random-effects models)

"The treatment works!"
(Alternative hypothesis models)

... YET ALL KEEP SHOUTING, TO SOME EXTENT.

RoBMA: Model Types

- Absence vs. presence of the:
 - Effect
 - Heterogeneity
 - Publication bias

THE PRIOR MODEL DEMONS ALL SHOUT ...



"Only sampling variability!"
(Fixed effects models)

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(Unadjusted models)

"No effect here!"

"Everything is biased!"
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"The treatment works!"
(Alternative hypothesis models)

THE PRIOR MODEL DEMONS ALL SHOUT ...



"How about direction of the effect?"

"Use PET model!"

"One-sided selection on significant p-values!"

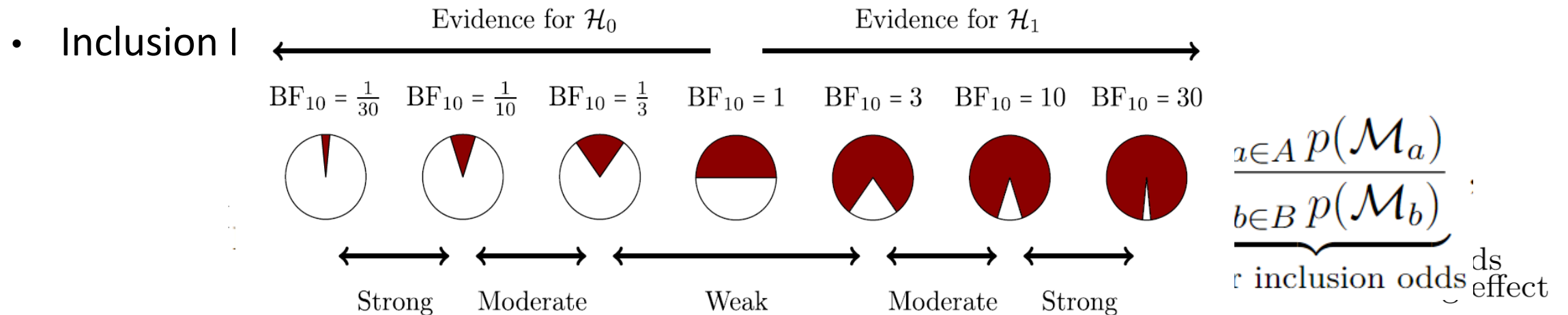
"The relationship with standard errors is quadratic!"

"Two-sided selection on significant and marginally significant p-values!"

RoBMA – Evaluating Evidence

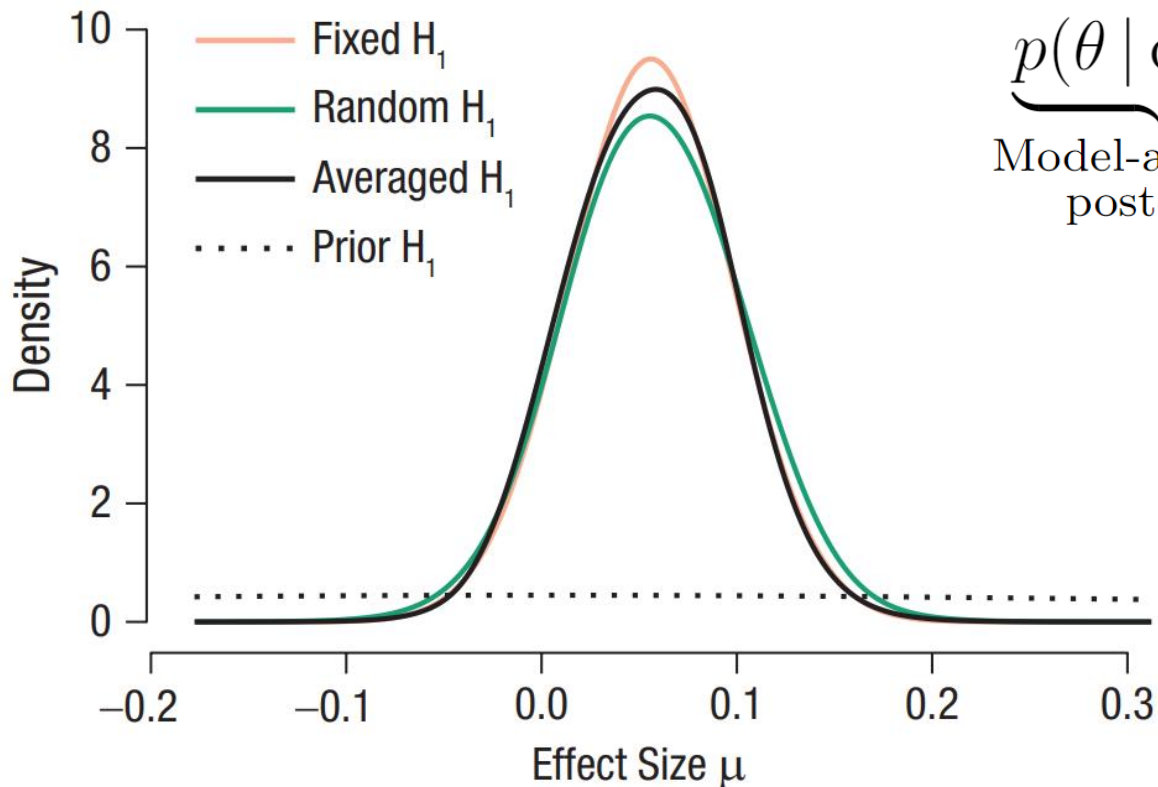
- Bayes factors quantify evidence for/against an effect/heterogeneity/publication bias:

$$BF_{10} = \frac{p(\text{data} \mid \mathcal{H}_1)}{p(\text{data} \mid \mathcal{H}_0)} = \underbrace{\frac{p(\text{data} \mid \mathcal{H}_1)}{p(\text{data} \mid \mathcal{H}_0)}}_{\text{Bayes factor}} = \underbrace{\frac{p(\mathcal{H}_1 \mid \text{data})}{p(\mathcal{H}_0 \mid \text{data})}}_{\text{Posterior odds}} \bigg/ \underbrace{\frac{p(\mathcal{H}_1)}{p(\mathcal{H}_0)}}_{\text{Prior odds}}$$



RoBMA – Estimating Parameters

- Model-averaged posterior distributions account for uncertainty in the selected models



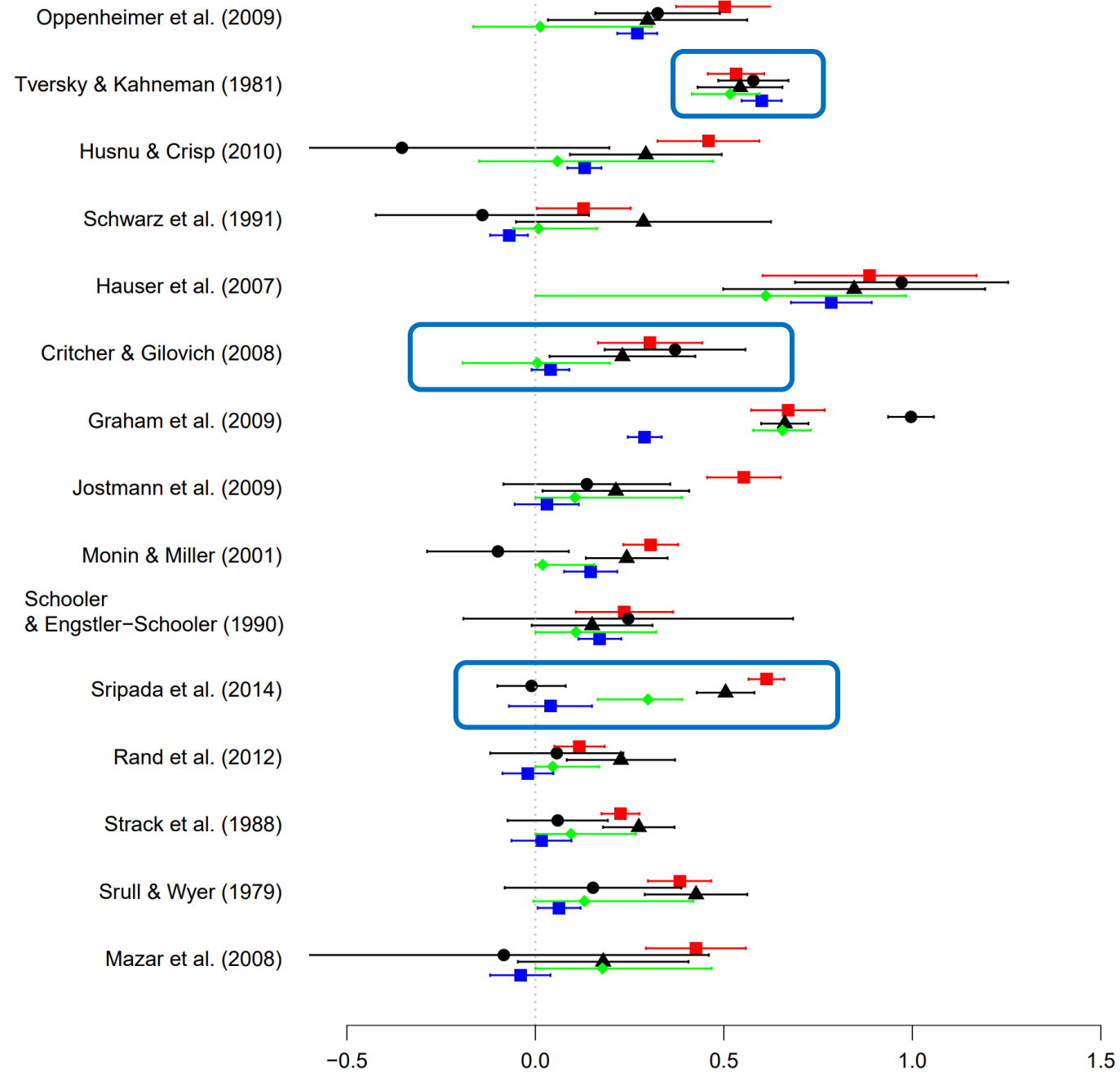
$$\underbrace{p(\theta | \text{data})}_{\text{Model-averaged posterior}} = \sum \underbrace{p(\theta | \mathcal{M}_., \text{data})}_{\text{Model specific posterior}} \underbrace{p(\mathcal{M}_. | \text{data})}_{\text{Posterior probability of model}}$$

RoBMA: Publication Bias Adjustment Components

- Models adjusting for relationship between effect sizes and standard errors
 - PET model (regression of effect sizes on standard errors)
 - PEESE model (regression of effect sizes on standard errors square)
- Selection models of p -values
 - Two-sided selection on significant p -values
 - Two-sided selection on significant and marginally significant p -values
 - One-sided selection on significant p -values
 - One-sided selection on significant and marginally significant p -values
 - One-sided selection on significant p -values and effects in expected direction
 - One-sided selection on significant, marginally significant p -values and effects in expected direction

Publication Bias Adjustment Methods

- Models adjusting for relationship between effect sizes and standard errors
 - Trim and fill (Duval & Tweedie, 2000)
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TABLE 2 Performance of 13 publication bias correction methods for the Kvarven and colleagues³⁴ test set comprised of 15 meta-analyses and 15 corresponding “Gold Standard” registered replication reports (RRR)

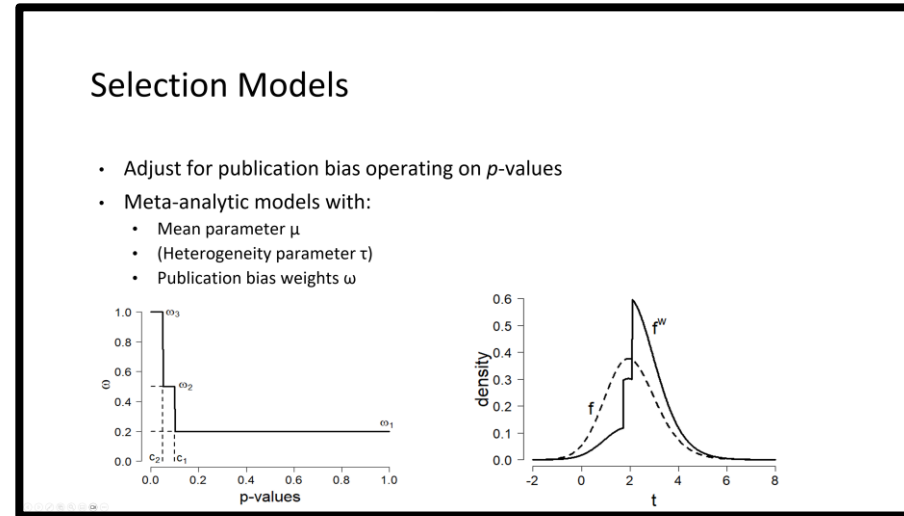
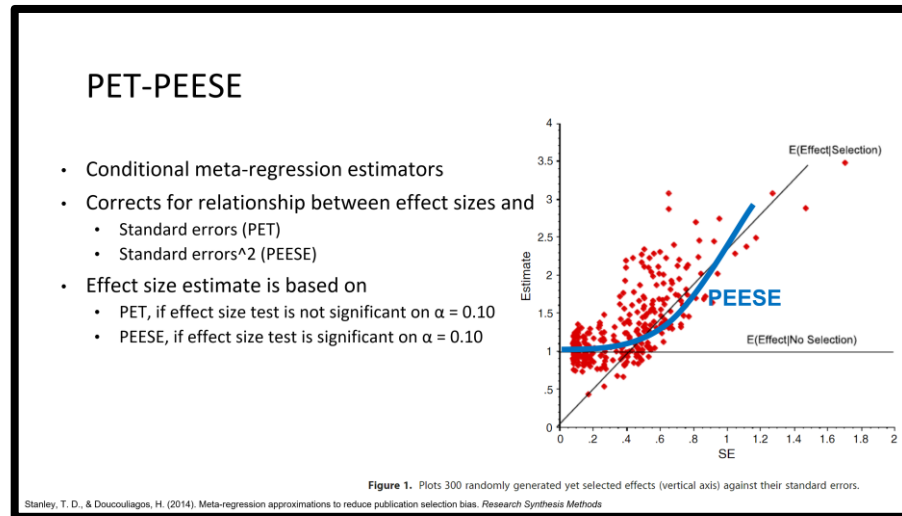
Method	FPR/Undecided	FNR/Undecided	OF	Bias	RMSE
RoBMA-PSMA	0.143/0.857	0.000/0.750	1.160	0.026	0.164
AK2	0.000/—	0.250/—	1.043	-0.070	0.268
PET-PEESE	0.143/—	0.500/—	1.307	0.050	0.256
EK	0.143/—	0.500/—	1.399	0.065	0.283
RoBMA-old	0.714/0.286	0.000/0.000	2.049	0.171	0.218
4PSM	0.714/—	0.500/—	1.778	0.127	0.268
3PSM	0.714/—	0.125/—	2.193	0.195	0.245
TF	0.833/—	0.000/—	2.315	0.206	0.259
AK1	0.857/—	0.000/—	2.352	0.221	0.264
<i>p</i> -uniform	0.500/—	0.429/—	2.375	0.225	0.288
<i>p</i> -curve			2.367	0.223	0.289
WAAP-WLS	0.857/—	0.125/—	2.463	0.239	0.295
Random Effects (DL)	1.000/—	0.000/—	2.586	0.259	0.310

Robust Bayesian Meta-Regression

- Extends RoBMA to moderators
- Bayesian model-averaging to base inference on multiple models simultaneously
- Accounts for uncertainty about the presence vs. absence of the effect/heterogeneity/publication bias/**moderators**
- Quantifies evidence in favor of the presence vs. absence of effect/heterogeneity/publication bias/**moderators**

Robust Bayesian Meta-Regression

- Uncertainty in model structure



- Under-powered moderation analyses

THE PRIOR MODEL DEMONS ALL SHOUT ...



'YOU NEED ONLY THIS PREDICTOR!'

'INCLUDE ALL PREDICTORS!'

'Include no predictor!'

THE PRIOR MODEL DEMONS ALL SHOUT ...



'YOU NEED ONLY THIS PREDICTOR!'

'INCLUDE ALL PREDICTORS'

THE PRIOR MODEL DEMONS ALL SHOUT ...



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(Fixed effects models)

"No bias here!"
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"Everything is biased!"



"The treatment works!"
(Alternative hypothesis models)

"There is additional heterogeneity!"
(Random effects models)

THE PRIOR MODEL DEMONS ALL SHOUT ...



"Use PET model!"

"How about direction of the effect?"

'YOU NEED "Use PEESE model!" PREDICTOR!'

"One-sided selection on significant p-values!"

"Two-sided selection on significant and marginally significant p-values!"

RoBMA.reg – Evaluating Evidence

$$\underbrace{\text{BF}_{\text{inclusion}}}_{\text{Inclusion Bayes factor}} = \frac{\sum_{a \in A} p(\mathcal{M}_a \mid \text{data})}{\underbrace{\sum_{b \in B} p(\mathcal{M}_b \mid \text{data})}_{\text{Posterior inclusion odds}}} \bigg/ \frac{\sum_{a \in A} p(\mathcal{M}_a)}{\underbrace{\sum_{b \in B} p(\mathcal{M}_b)}_{\text{Prior inclusion odds}}}$$

- Inclusion Bayes factors for the moderation effect

$$\underbrace{\text{BF}_{\text{moderation}}}_{\substack{\text{Inclusion Bayes factor} \\ \text{for moderation}}} = \frac{\sum_{a \in A} p(\mathcal{M}_a \mid \text{data})}{\underbrace{\sum_{b \in B} p(\mathcal{M}_b \mid \text{data})}_{\substack{\text{Posterior inclusion odds} \\ \text{for moderation}}}} \bigg/ \frac{\sum_{a \in A} p(\mathcal{M}_a)}{\underbrace{\sum_{b \in B} p(\mathcal{M}_b)}_{\substack{\text{Prior inclusion odds} \\ \text{for moderation}}}}$$

RoBMA.reg – Estimating Parameters

- Model-averaged posterior distributions account for uncertainty in the selected models

$$\underbrace{p(\theta | \text{data})}_{\text{Model-averaged posterior}} = \sum \underbrace{p(\theta | \mathcal{M}_., \text{data})}_{\text{Model specific posterior}} \underbrace{p(\mathcal{M}_. | \text{data})}_{\text{Posterior probability of model}}$$

Some Complications

- Parameterization
- Follow-up analyses

Continuous vs. Categorical Moderators

- Different scaling (continuous moderators) and contrasts coding (factor moderators) corresponds to different hypotheses

Continuous moderators

- Centering
 - => intercept corresponds to the mean effect
(prior distribution on the mean effect corresponds to a meta-analysis)
- Scaling
 - => standardized meta-regression coefficients
(prior distribution on the regression coefficient is scale invariant)

Continuous vs. Categorical Moderators

Categorical moderators

- Dummy coding
 - => intercept corresponds to the effect in the default category
 - => individual dummy coefficients test for differences between the default and remaining categories
- (Scaled) Orthonormal contrasts
 - => intercept corresponds to the mean effect
(prior distribution on the mean effect corresponds to a meta-analysis)
 - => individual orthonormal coefficients on the differences of each category and the mean effect
(prior distribution on the regression coefficients is label invariant)

Continuous vs. Categorical Moderators

Default parameter prior distributions (Cohen's d)

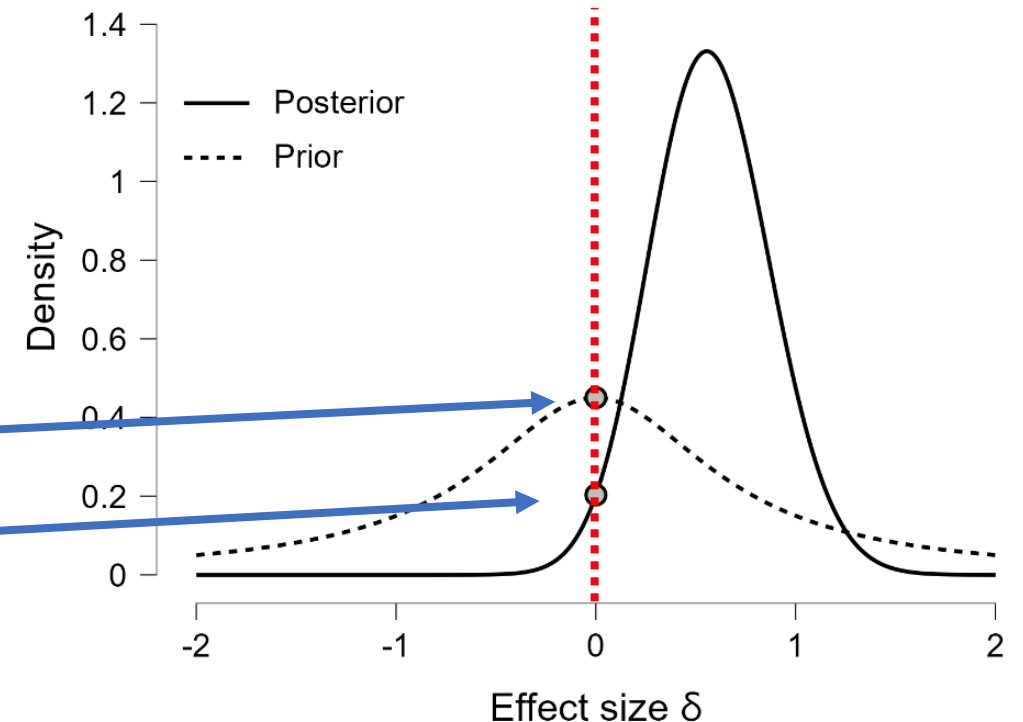
- Standard normal prior distribution on the mean effect
- Normal prior distribution with mean = 0 and standard deviation = $\frac{1}{4}$ on centered and scaled continuous moderators
- Normal prior distribution with mean = 0 and standard deviation = $\frac{1}{4}$ on differences from grand mean for each factor level via scaled orthonormal contrasts

Testing Subgroup Effects

- Categorical predictors: “Is there an effect in group A?”
 - Subgroup analyses (data sub-setting)
 - Savage Dickey density ratio with model-averaged prior/posterior distributions (assuming presence of the effect or moderation)

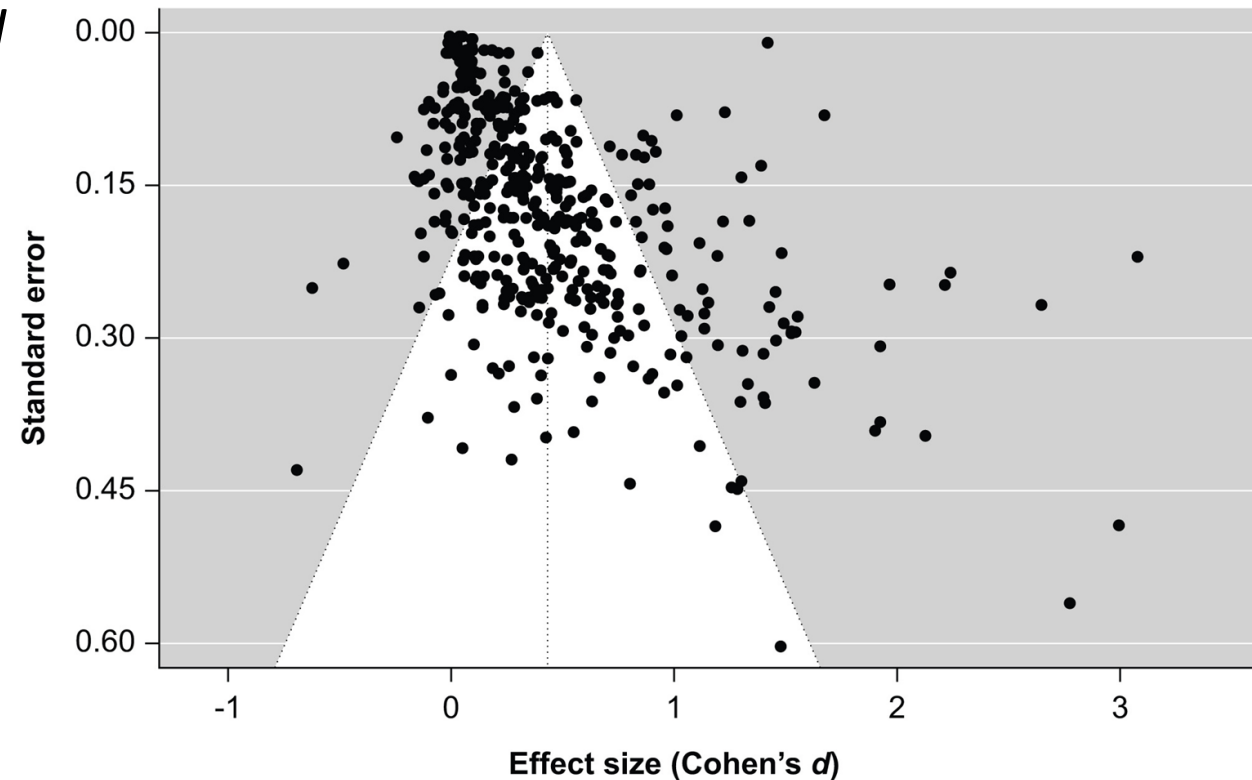
$$BF_{10} = \frac{p(\text{data} \mid \mathcal{H}_1)}{p(\text{data} \mid \mathcal{H}_0)}$$

$$BF_{10} = \frac{p(\theta_g = 0 \mid A)}{p(\theta_g = 0 \mid \text{data}, A)}$$



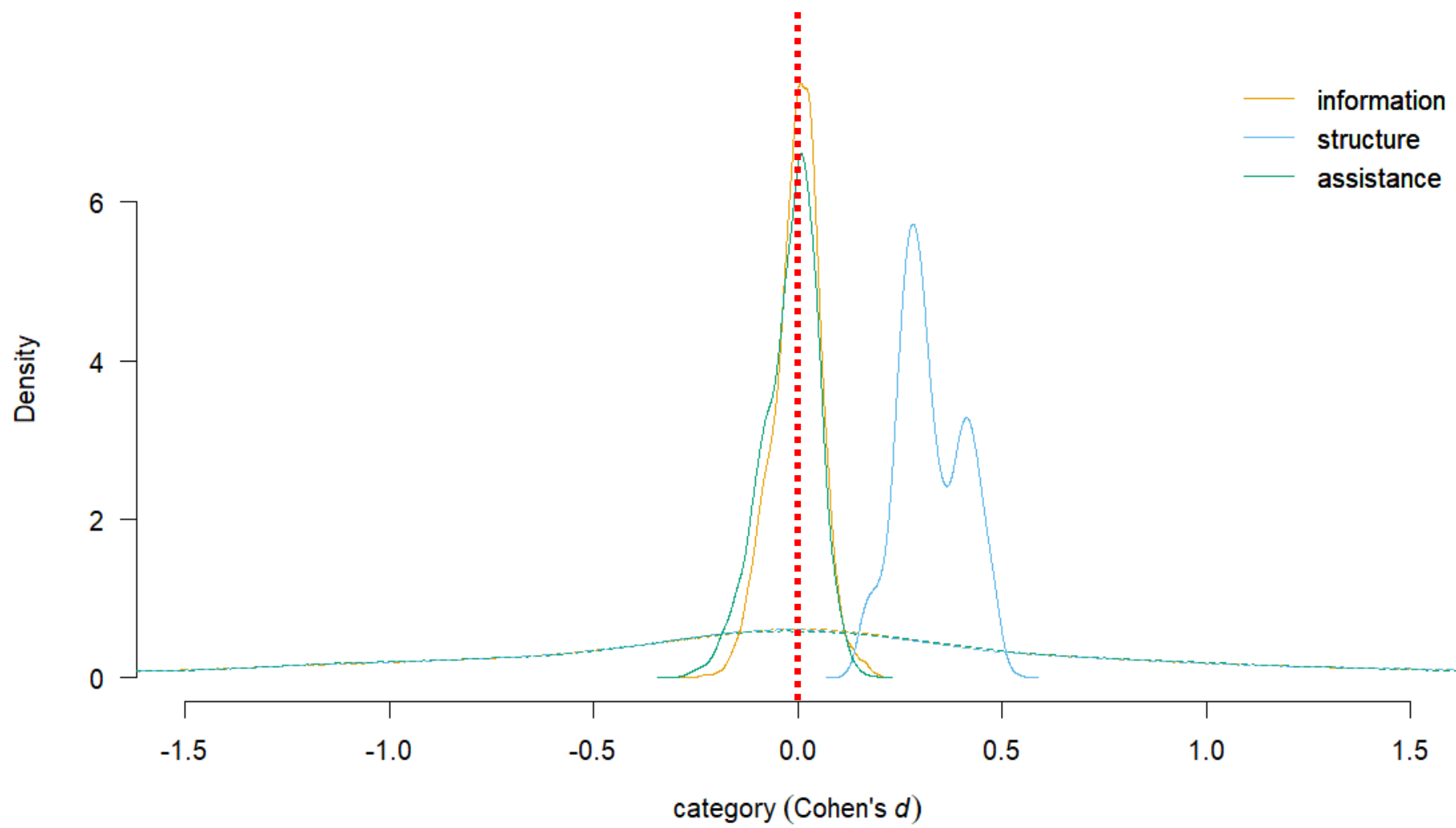
Example: No Evidence for Nudging

- Mertens and colleagues (2022) conducted large meta-analysis on nudging
“choice architecture is an effective and widely applicable behaviour change tool” (p. 8)
- Effect moderated based on domain and category of nudge



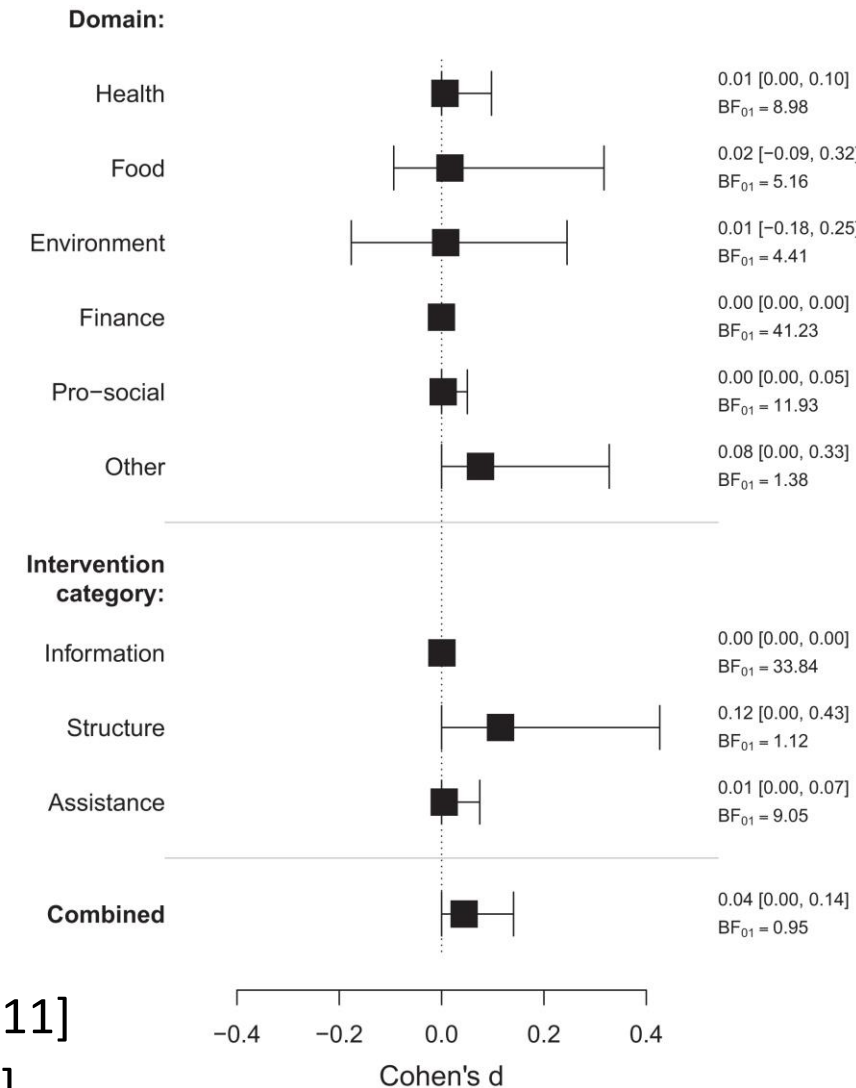
Example: RoBMA

- Effect: $BF_{10} = 1.20$, $\mu = 0.06$, 95% CI [0.00, 0.17]
- Heterogeneity: $BF_{rf} = \text{Inf}$, $\tau = 0.36$, 95% CI [0.27, 0.45]
- Publication Bias: $BF_{pb} = 1.02 \times 10^{13}$
- Moderation
 - Domain: $BF_{10} = 2.33$
 - Category: $BF_{10} = 1.60 \times 10^{11}$
- Subgroups by *Category*
 - Information: $BF_{10} = 0.08$, $\mu_{\text{information}} = 0.00$, 95% CI [-0.13, 0.11]
 - Structure: $BF_{10} = \text{Inf}$, $\mu_{\text{structure}} = 0.32$, 95% CI [0.17, 0.48]
 - Assistance: $BF_{10} = 0.09$, $\mu_{\text{assistance}} = 0.02$, 95% CI [-0.18, 0.10]



Example: RoBMA

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 - Assistance: $BF_{10} = 0.09$, $\mu_{\text{assistance}} = 0.02$, 95% CI [-0.18, 0.10]



Simulation Study

1. $\mu = (0, 0.2, 0.5)$
2. $\beta = (0, 0.2, 0.5)$
3. $\tau = (0, 0.2, 0.4)$
4. $K = (30, 100)$
5. Publication bias
 - a. No bias: $\omega_1 = 1, \omega_2 = 1, \omega_3 = 1$
 - b. Moderate bias: $\omega_1 = 0.2, \omega_2 = 0.5, \omega_3 = 1$
 - c. Strong bias: $\omega_1 = 0, \omega_2 = 0, \omega_3 = 1$

ω_1 = Nonsignificant studies

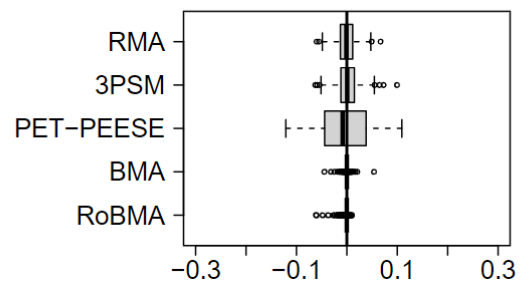
ω_2 = Marginally significant studies

ω_3 = Significant studies

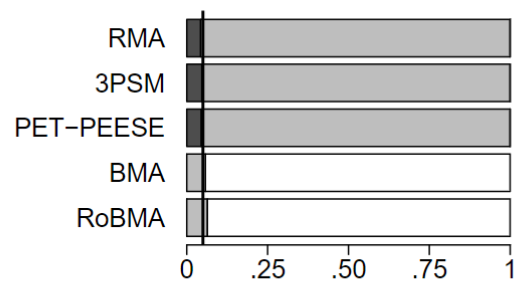
Simulation Study Results (Select Cases)

No effect ($\mu = 0$)
 No moderation ($\beta = 0$)
 No heterogeneity ($\tau = 0$)
 No pub. bias

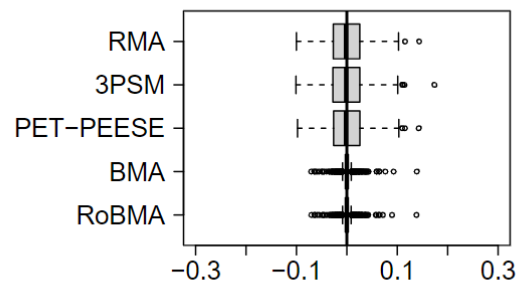
μ (estimate)



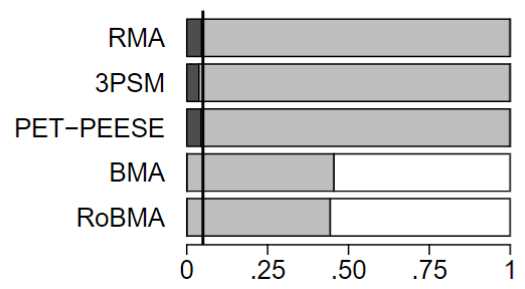
μ (test)



β (estimate)



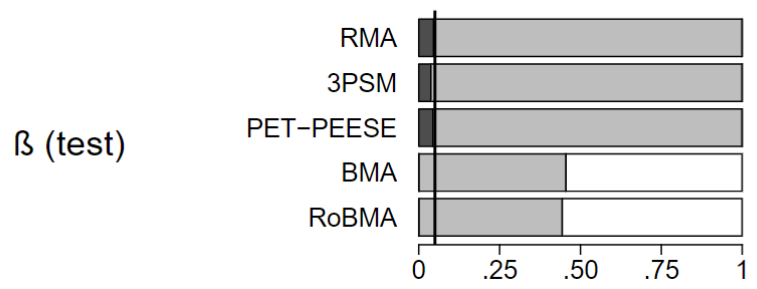
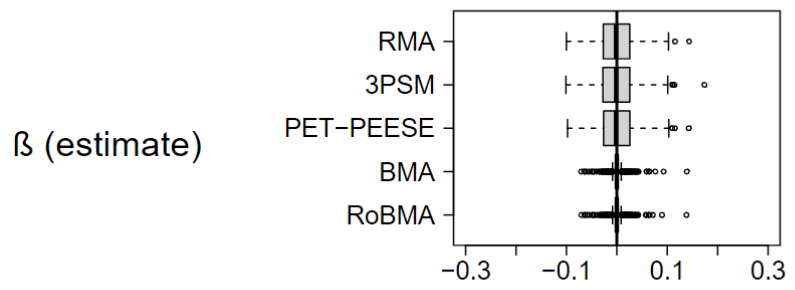
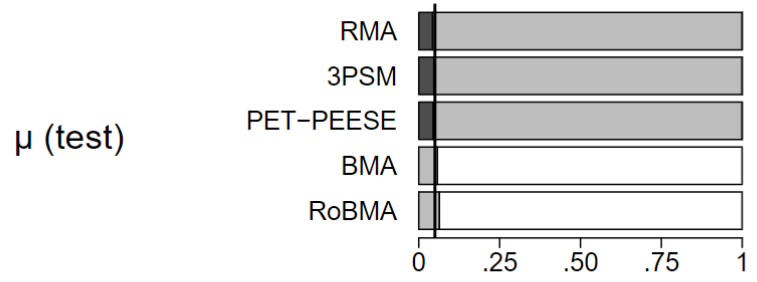
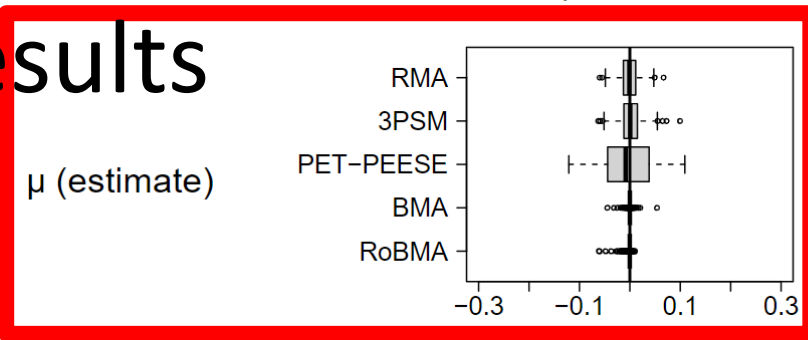
β (test)



Simulation Study Results

(Select Cases)

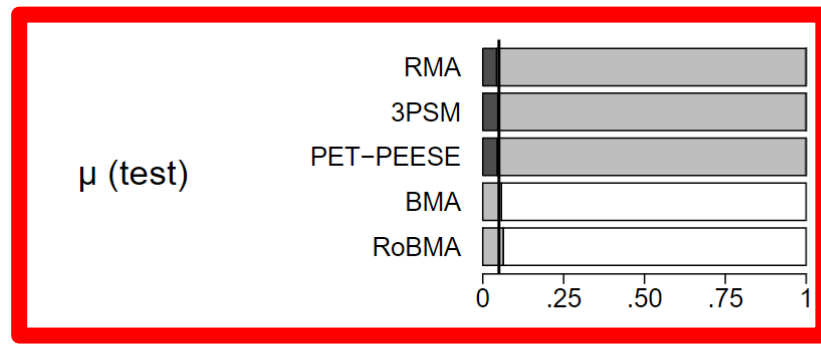
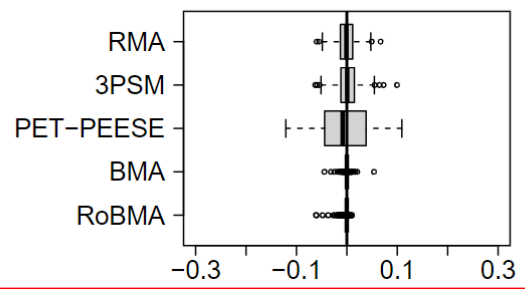
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 No pub. bias



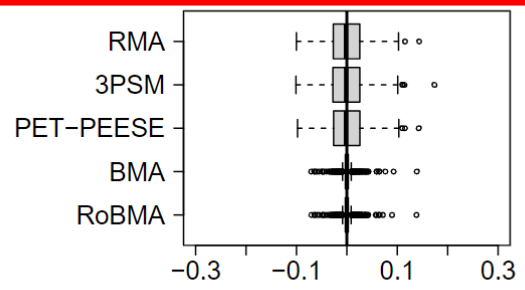
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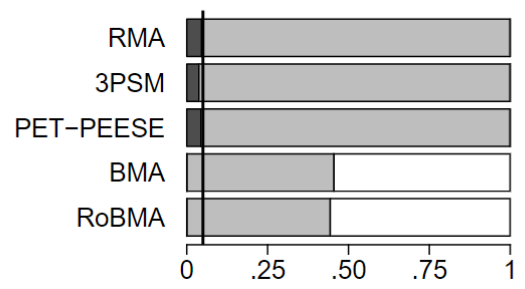
μ (estimate)



β (estimate)



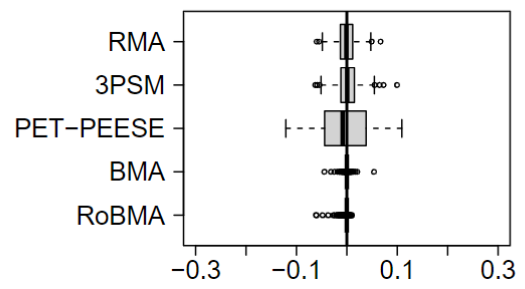
β (test)



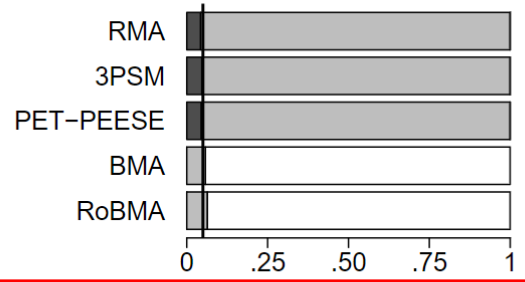
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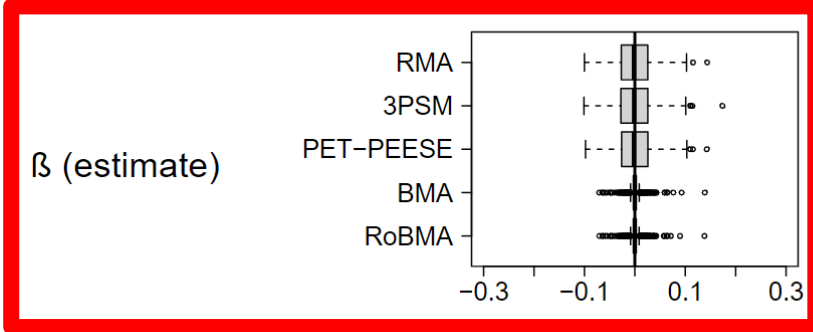
μ (estimate)



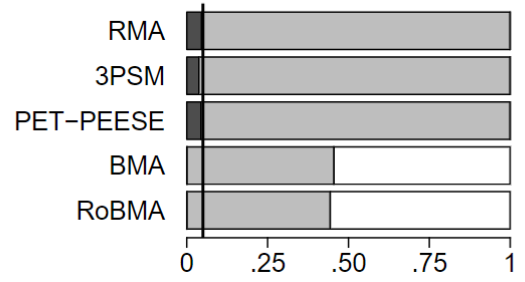
μ (test)



β (estimate)



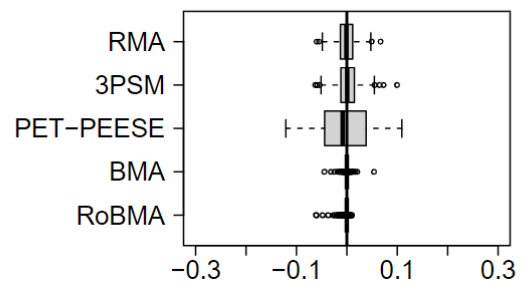
β (test)



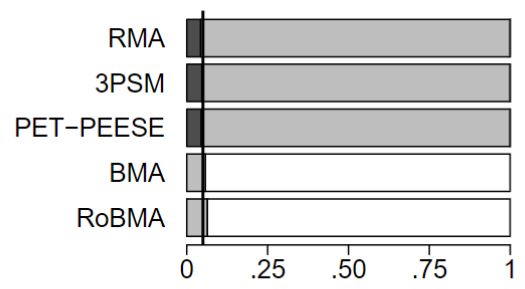
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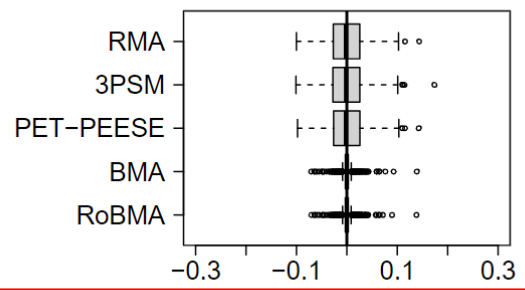
μ (estimate)



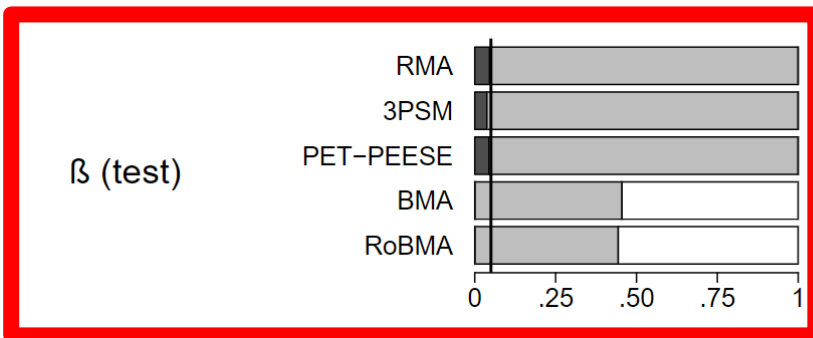
μ (test)



β (estimate)

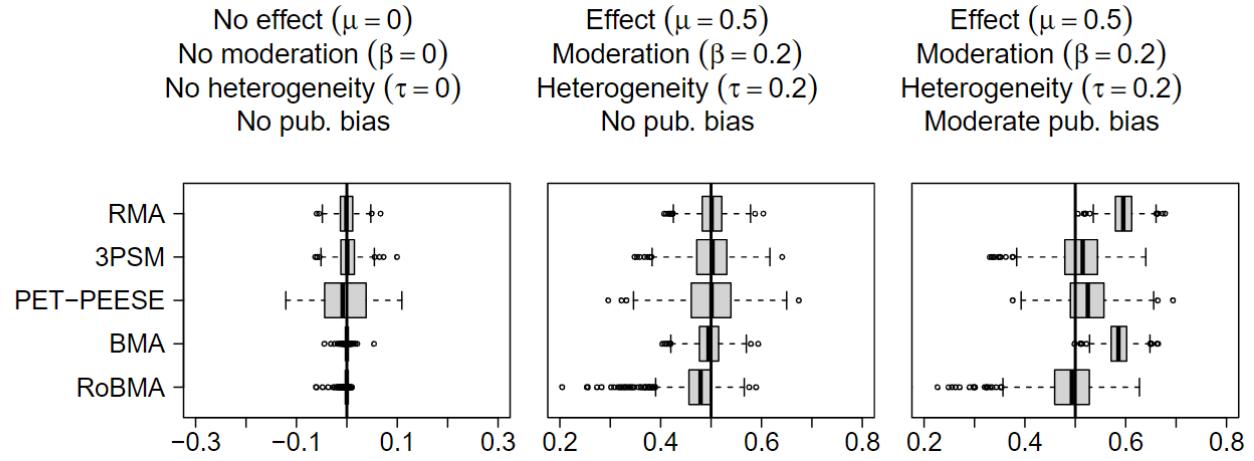


β (test)

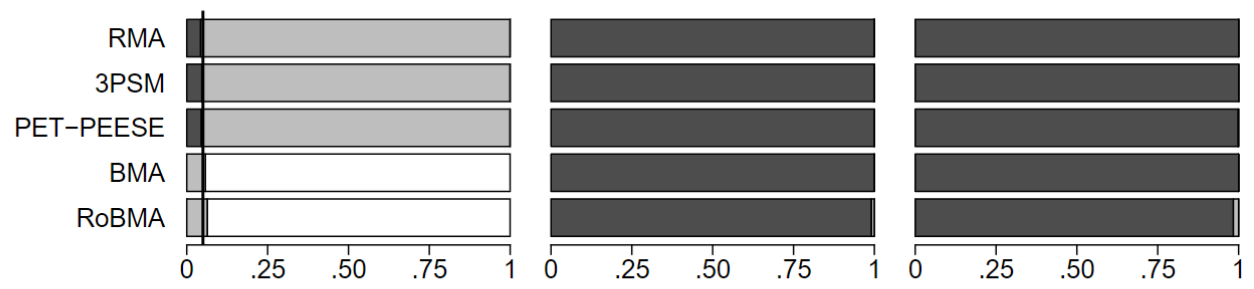


Simulation Study Results (Select Cases)

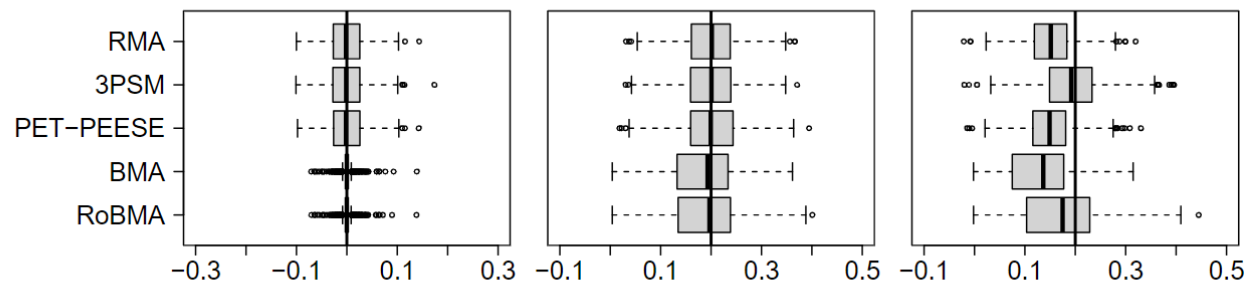
μ (estimate)



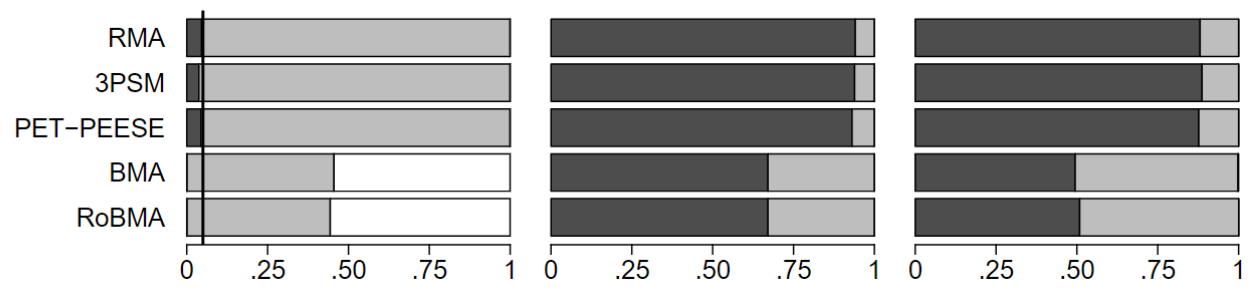
μ (test)



β (estimate)

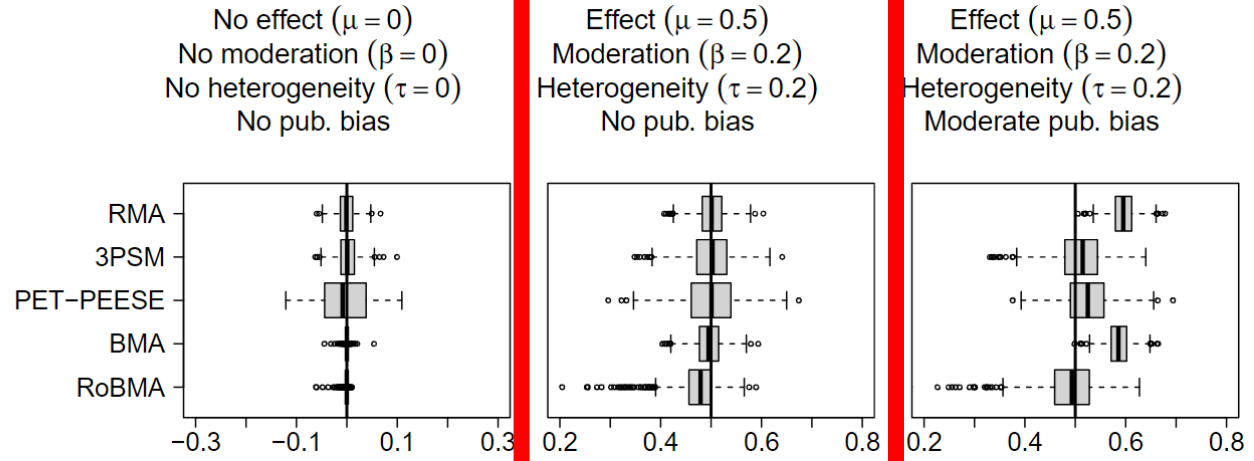


β (test)

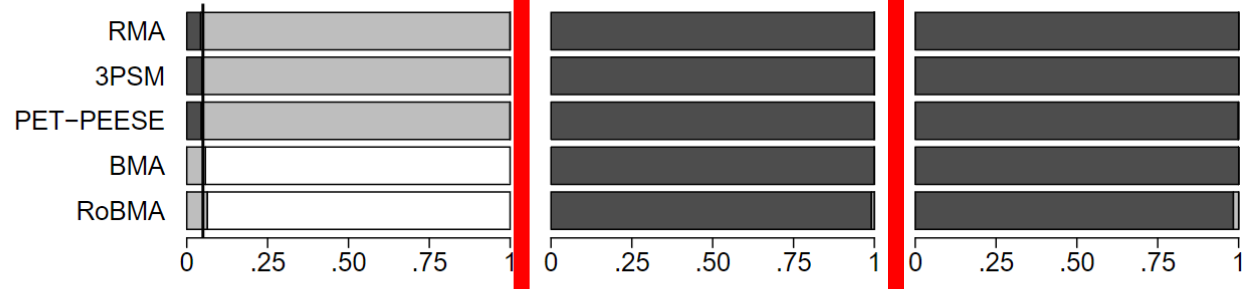


Simulation Study Results (Select Cases)

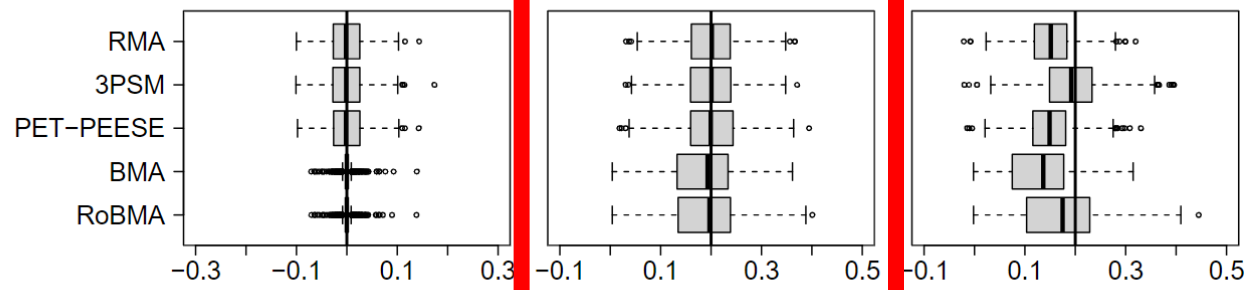
μ (estimate)



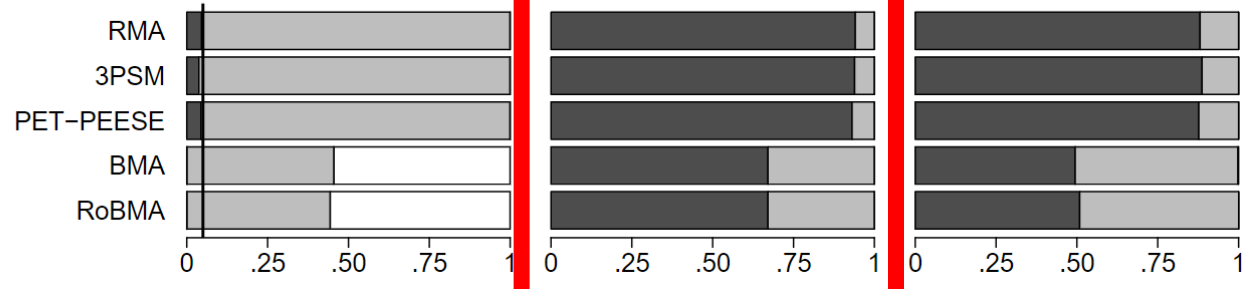
μ (test)



β (estimate)

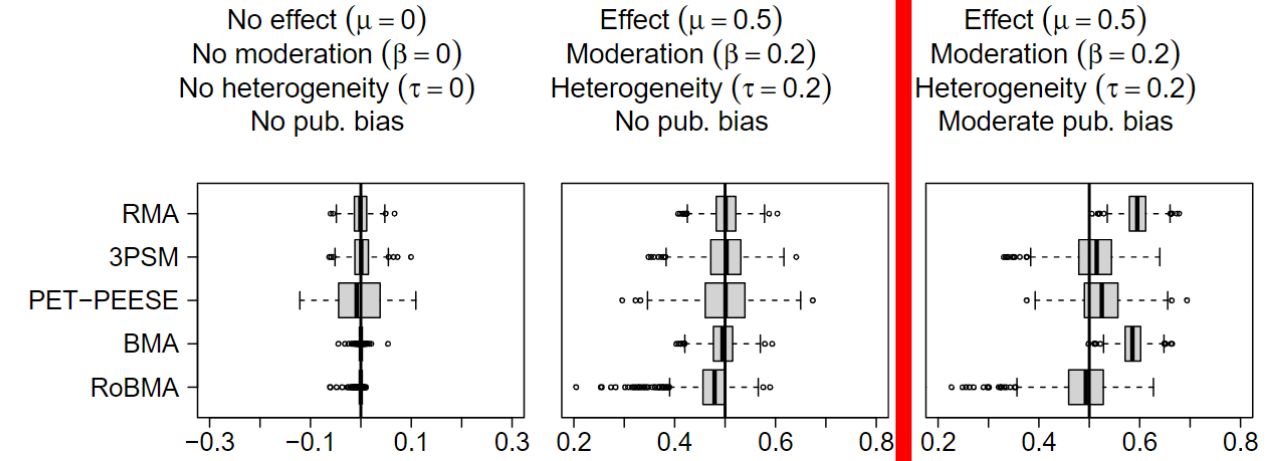


β (test)

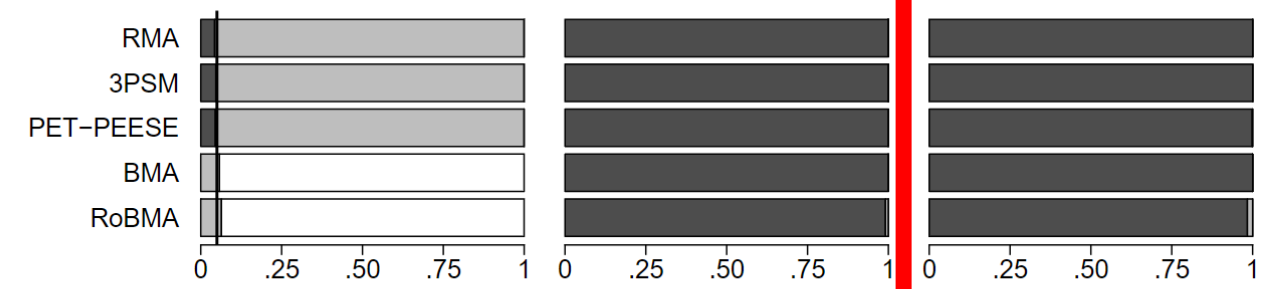


Simulation Study Results (Select Cases)

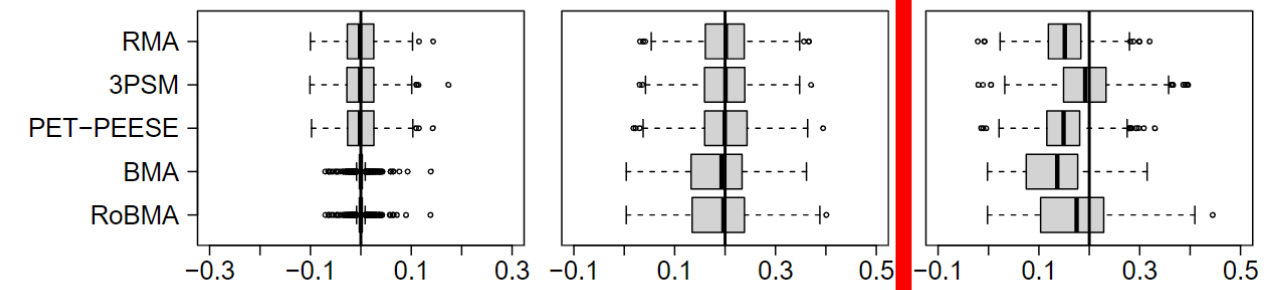
μ (estimate)



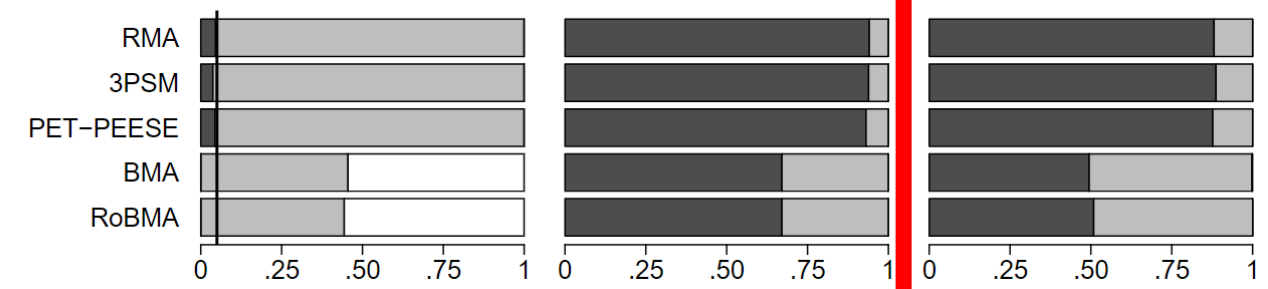
μ (test)



β (estimate)

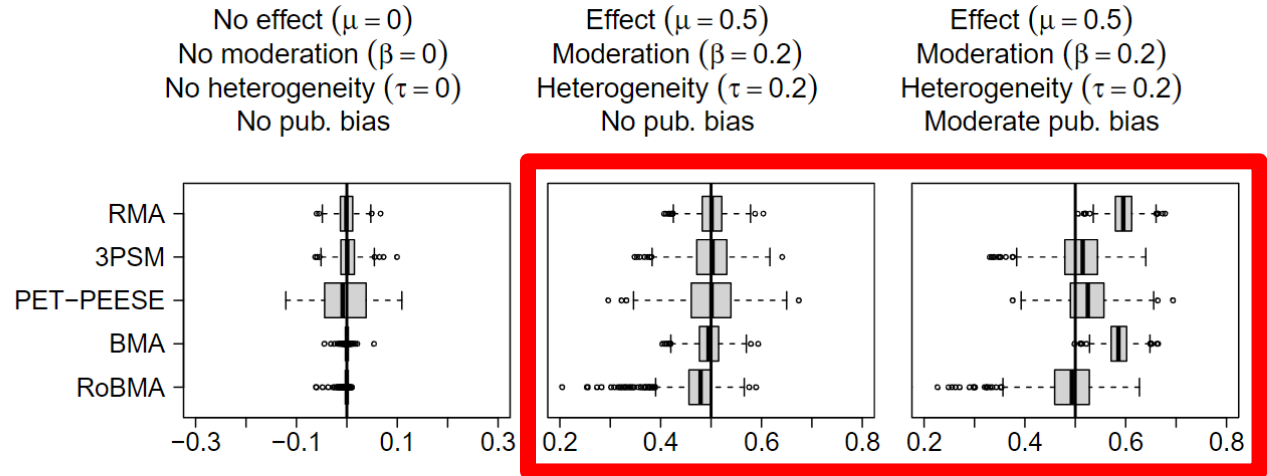


β (test)

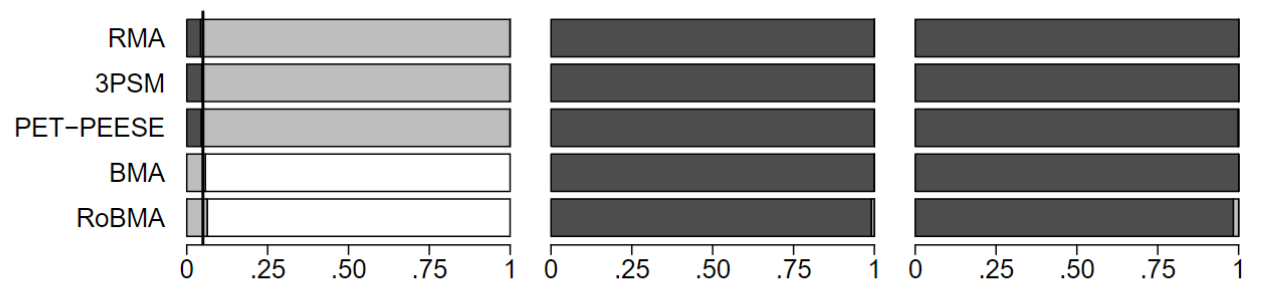


Simulation Study Results (Select Cases)

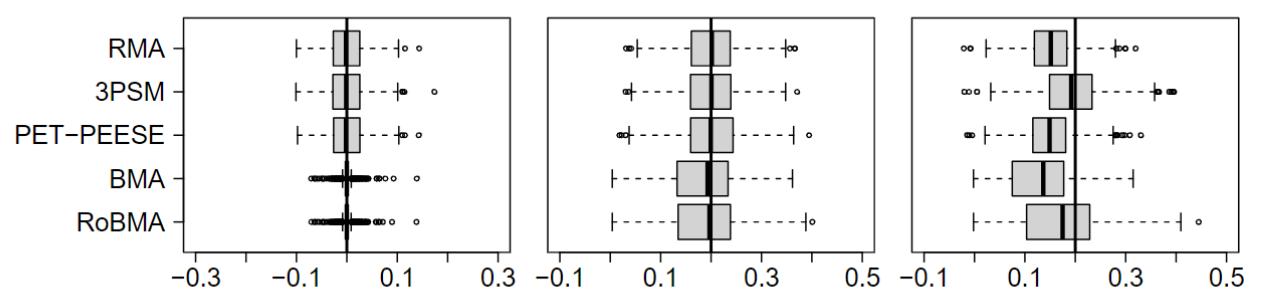
μ (estimate)



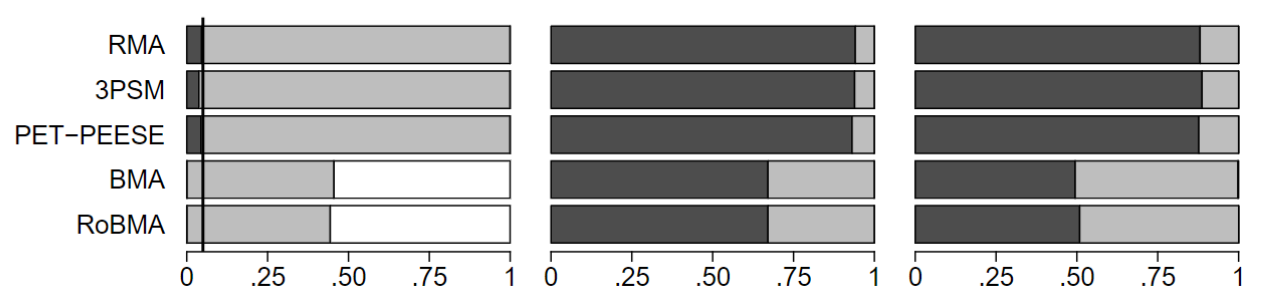
μ (test)



β (estimate)

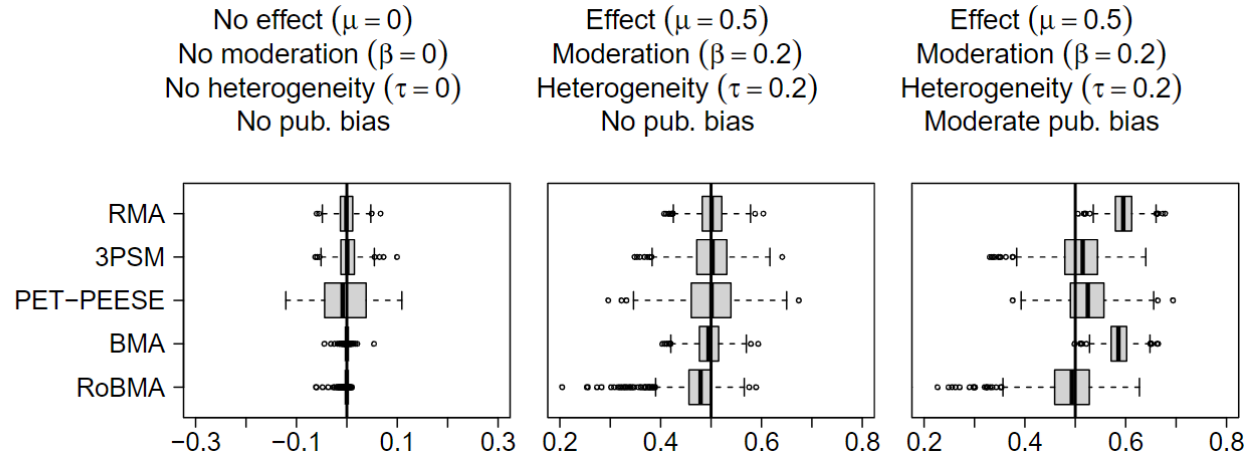


β (test)

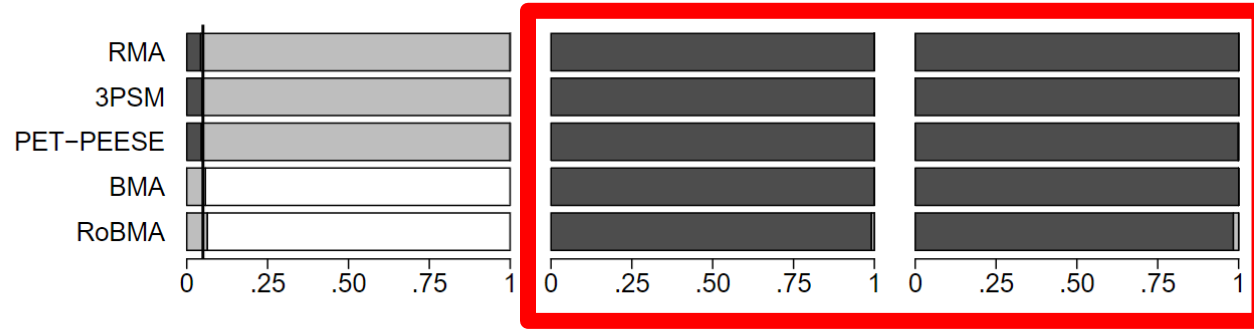


Simulation Study Results (Select Cases)

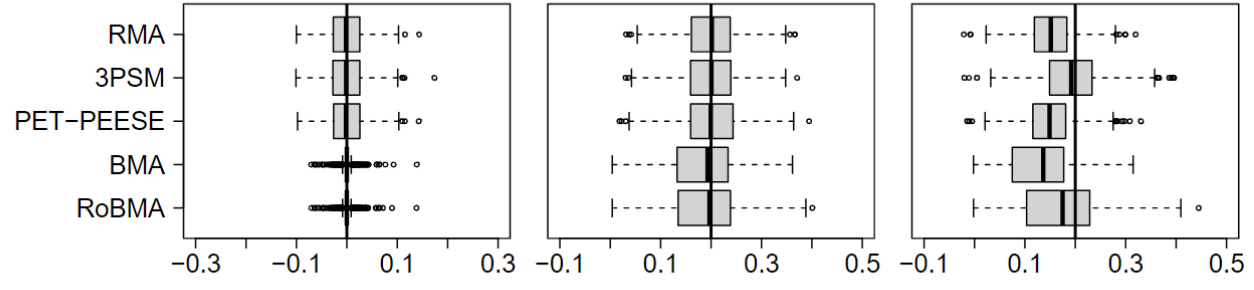
μ (estimate)



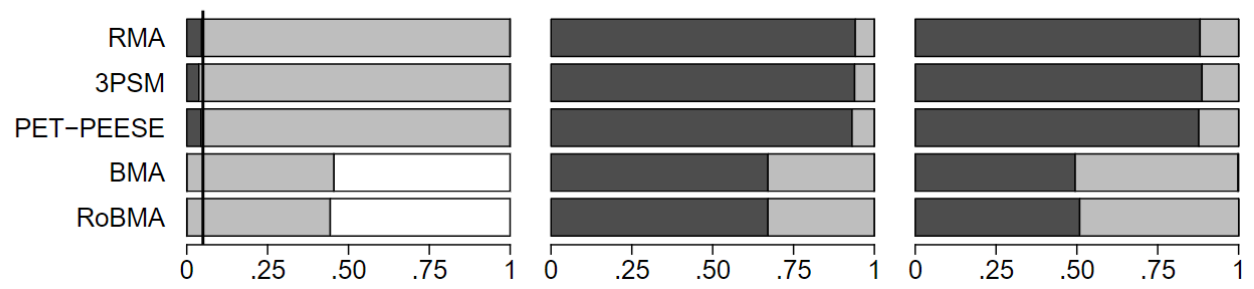
μ (test)



β (estimate)

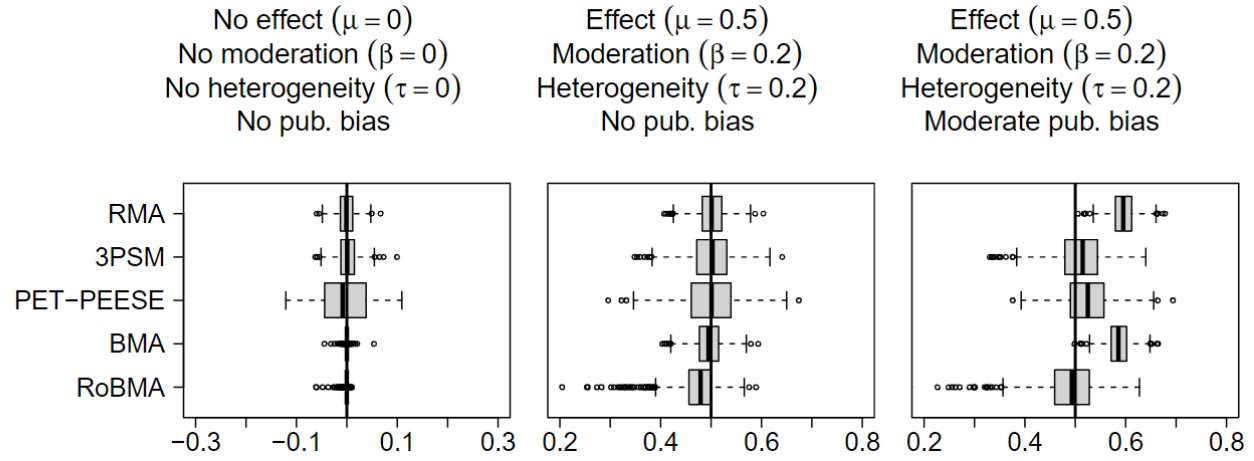


β (test)

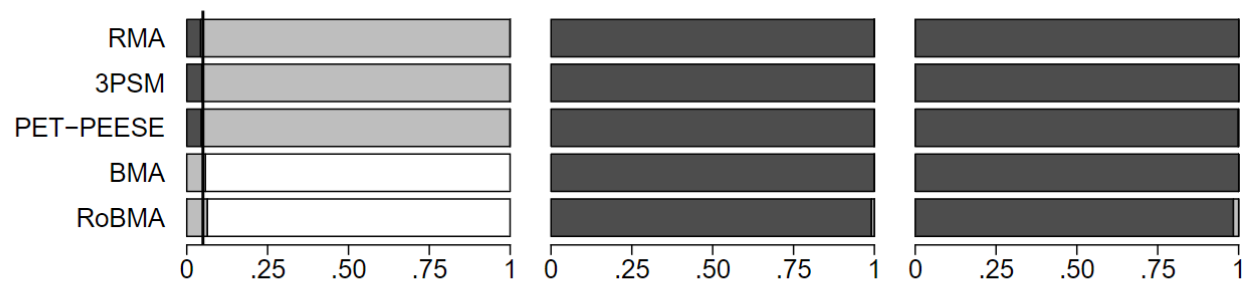


Simulation Study Results (Select Cases)

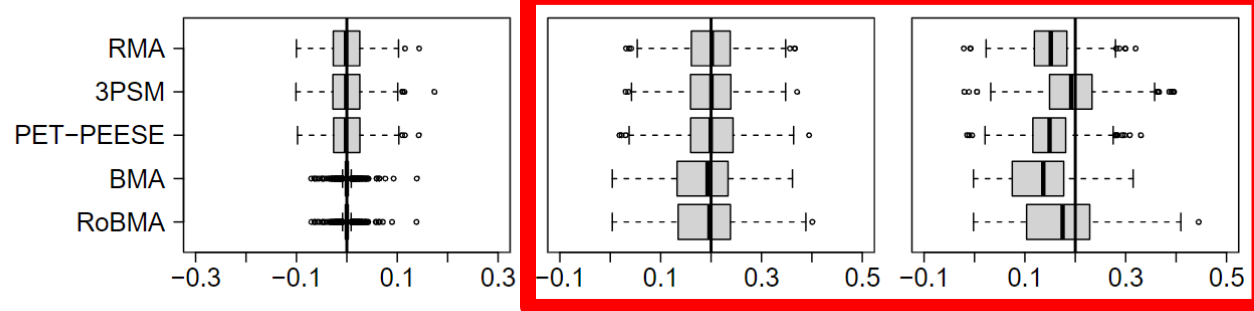
μ (estimate)



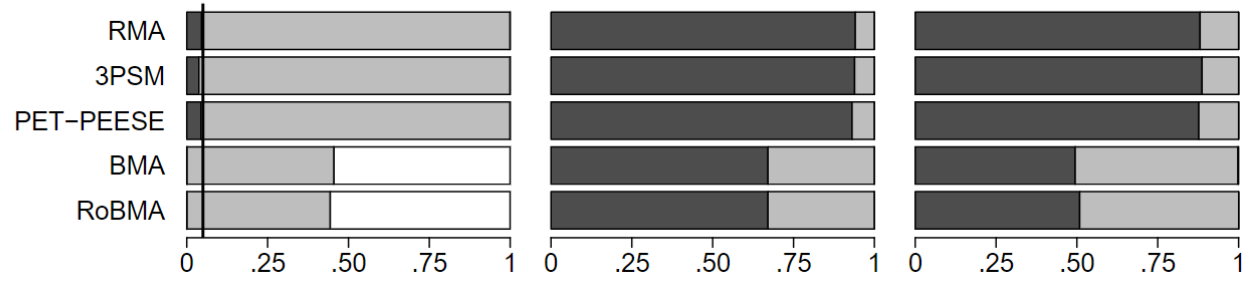
μ (test)



β (estimate)

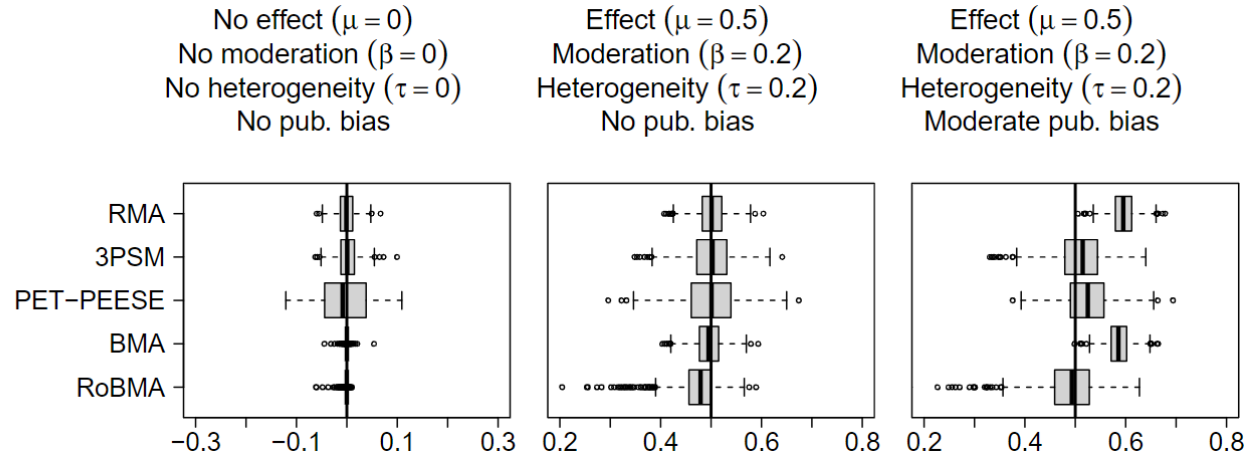


β (test)

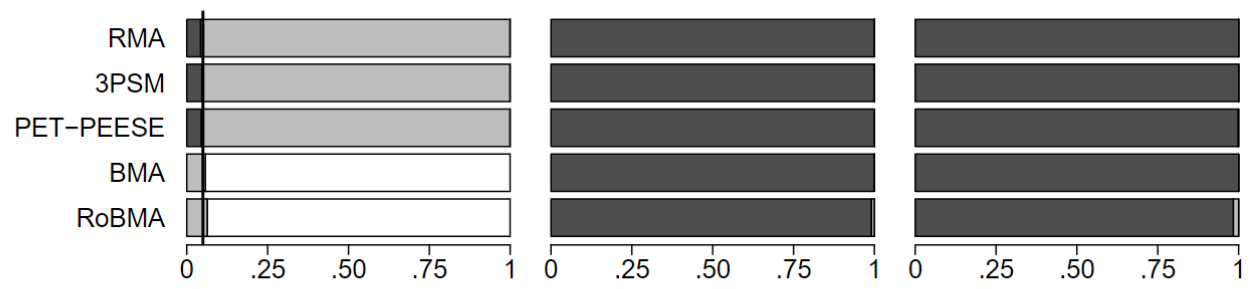


Simulation Study Results (Select Cases)

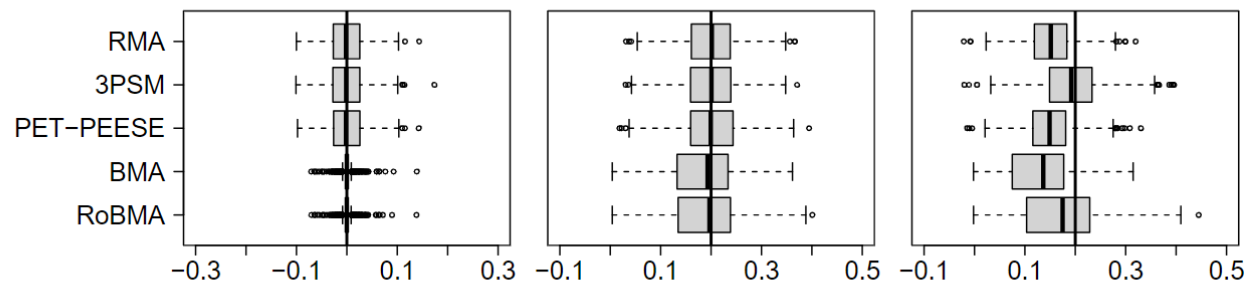
μ (estimate)



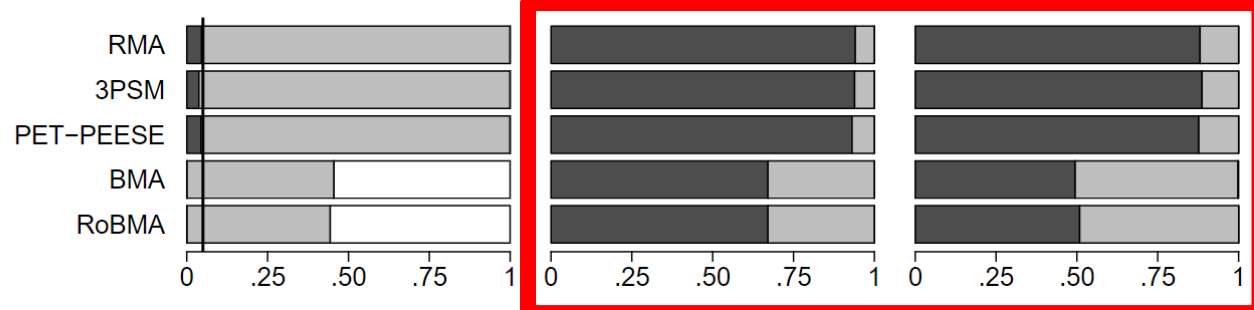
μ (test)



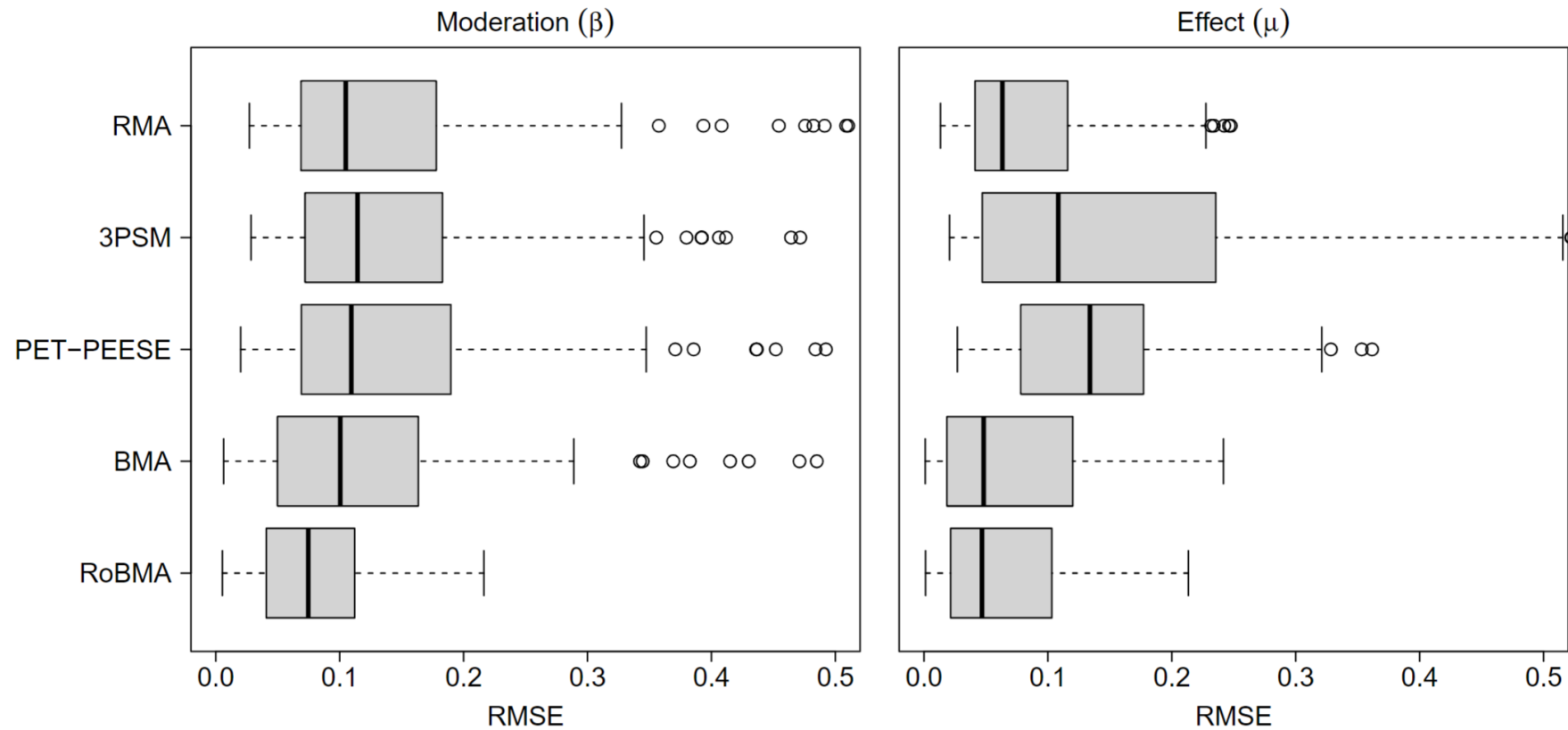
β (estimate)



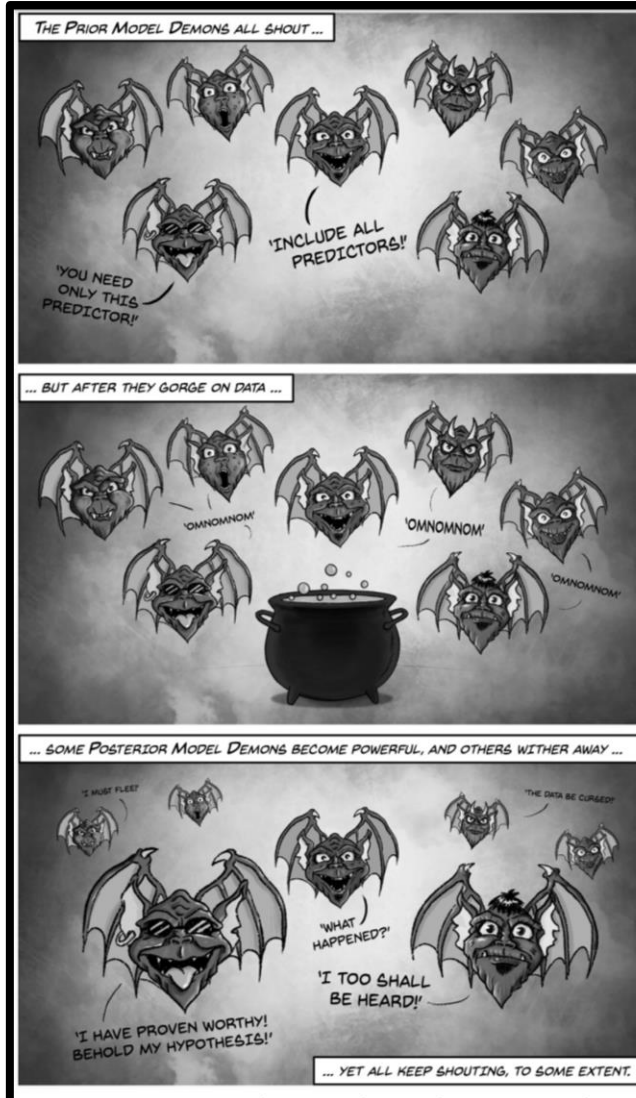
β (test)



Simulation Study Results (Across Conditions)



How to Run RoBMA



prior parameter distributions → Specify the

$$p(\Theta_0 | \mathcal{H}_0^f) : \mu = 0$$

$$p(\text{data} | \Theta_0, \mathcal{H}_0^f) : y \sim \text{Normal}(\mu, \text{se})$$

Update the

posterior parameter distributions

$$p(\Theta_0 | \mathcal{H}_0^f, \text{data}) = \frac{p(\text{data} | \Theta_0, \mathcal{H}_0^f) p(\Theta_0 | \mathcal{H}_0^f)}{p(\text{data} | \mathcal{H}_0^f)}$$

$$p(\Theta_1 | \mathcal{H}_1^f, \text{data}) = \frac{p(\text{data} | \Theta_1, \mathcal{H}_1^f) p(\Theta_1 | \mathcal{H}_1^f)}{p(\text{data} | \mathcal{H}_1^f)}$$

marginal likelihoods

$$p(\text{data} | \mathcal{H}_0^f) = \int p(\text{data} | \Theta_0, \mathcal{H}_0^f) p(\Theta_0 | \mathcal{H}_0^f) d\Theta_0$$

$$p(\text{data} | \mathcal{H}_1^f) = \int p(\text{data} | \Theta_1, \mathcal{H}_1^f) p(\Theta_1 | \mathcal{H}_1^f) d\Theta_1$$

Bayes factors →

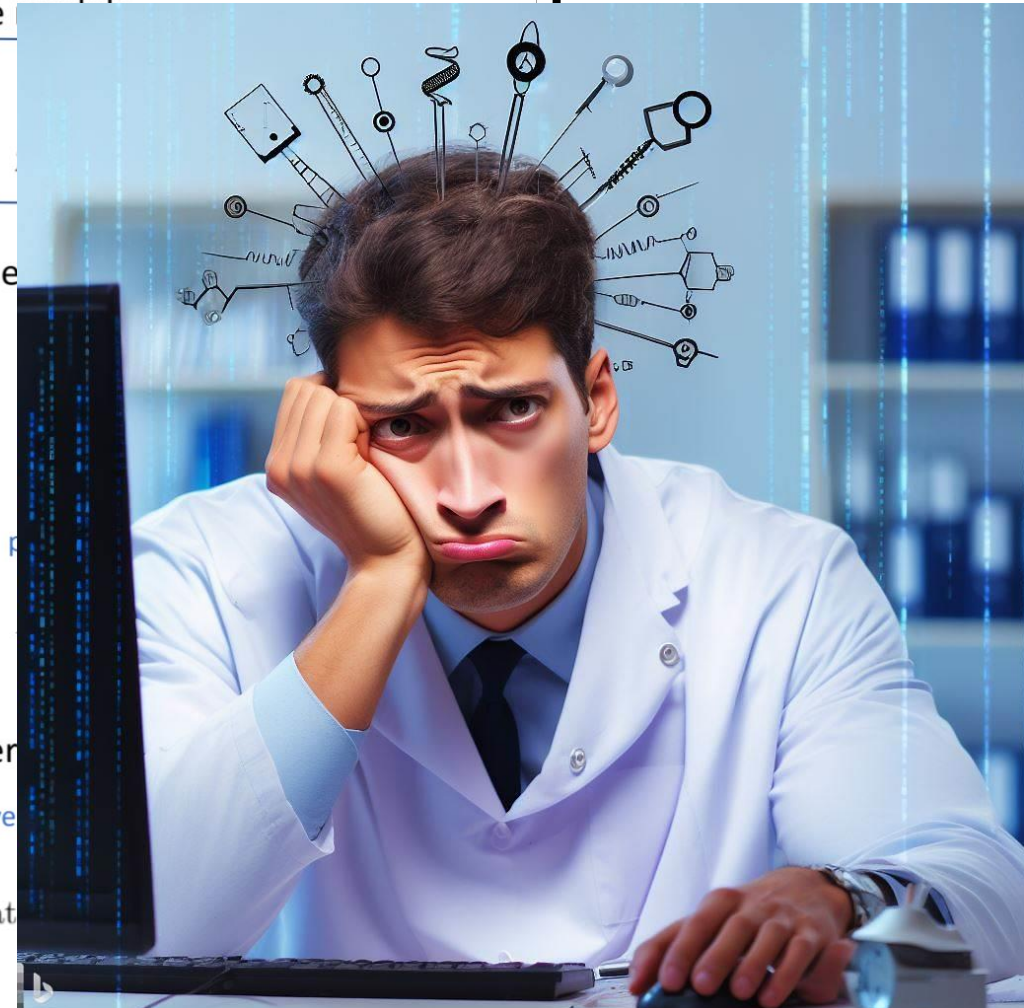
$$\text{BF}_{10} = \frac{p(\text{data} | \mathcal{H}_1^f)}{p(\text{data} | \mathcal{H}_0^f)}$$

$$\underbrace{\frac{p(\text{data} | \mathcal{H}_1^f)}{p(\text{data} | \mathcal{H}_0^f)}}_{\text{Bayes factor}} = \underbrace{\frac{p(\mathcal{H}_1^f | \text{data})}{p(\mathcal{H}_0^f | \text{data})}}_{\text{Posterior odds}} \bigg/ \underbrace{\frac{p(\mathcal{H}_1^f)}{p(\mathcal{H}_0^f)}}_{\text{Prior odds}}$$

Draw infer

Model-ave

$$p(\Theta | \text{data})$$



RoBMA Implementation (R)

```
library(RoBMA)

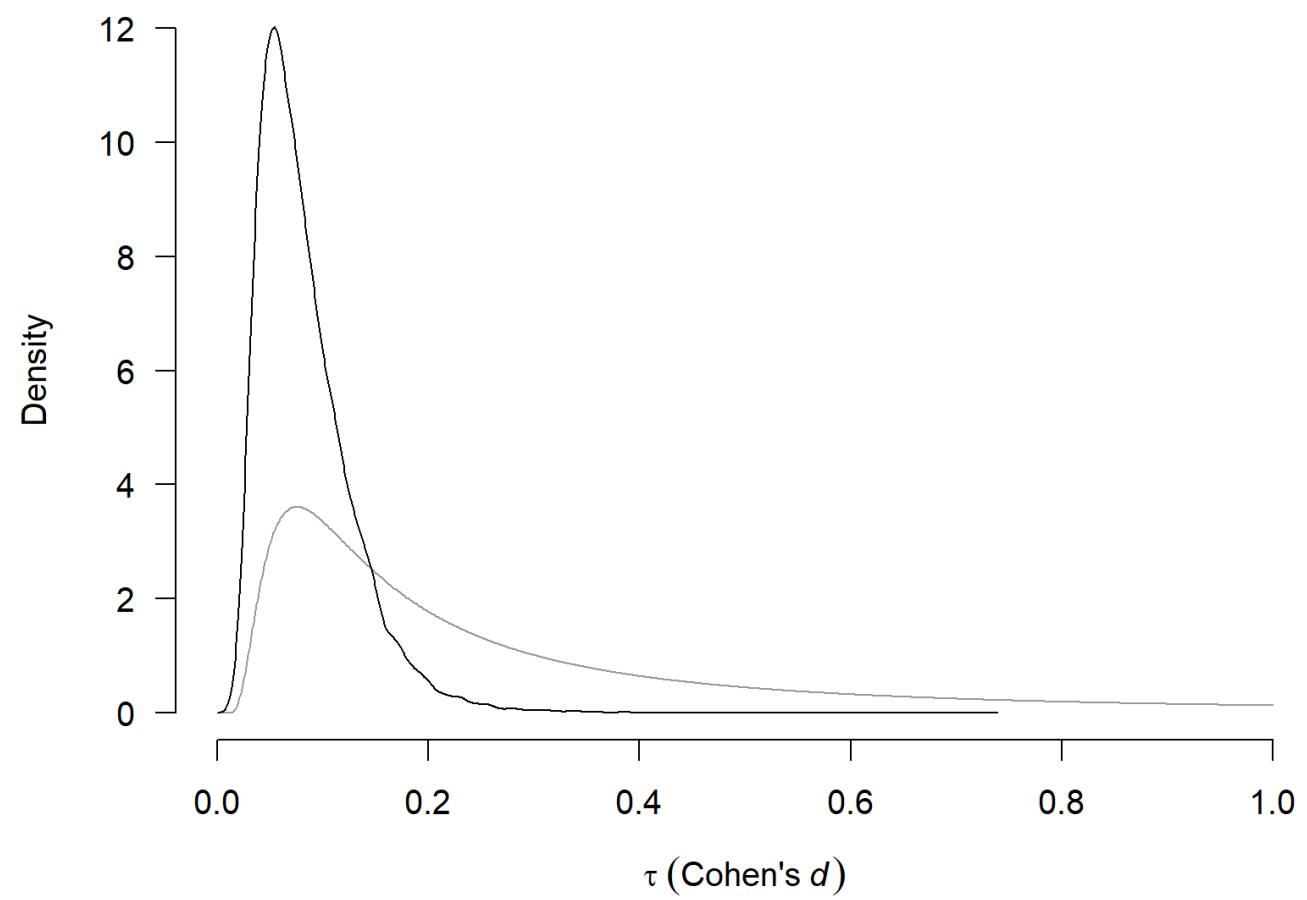
fit <- RoBMA(d = Bem2011$d, se = Bem2011$se)

summary(fit)

> Robust Bayesian Meta-Analysis
>
>
>           Models Prior prob. Post. prob. Incl. BF
> Effect      18/36      0.500      0.324      0.480
> Heterogeneity 18/36      0.500      0.125      0.143
> Pub. bias    32/36      0.500      0.942     16.297

> Model-averaged estimates
>
>           Mean Median 0.025 0.975
> mu          0.037 0.000 -0.051 0.218
> tau         0.010 0.000 0.000 0.113
> omega[0,0.025] 1.000 1.000 1.000 1.000
> omega[0.025,0.05] 0.934 1.000 0.332 1.000
> omega[0.05,0.5] 0.784 1.000 0.009 1.000
> omega[0.5,0.95] 0.771 1.000 0.007 1.000
> omega[0.95,0.975] 0.787 1.000 0.007 1.000
> omega[0.975,1] 0.803 1.000 0.007 1.000
> PET         0.758 0.000 0.000 2.790
> PEESE       6.222 0.000 0.000 25.597
```

```
fit <- RoBMA(d = Bem2011$d, se = Bem2011$se)
plot(fit)
plot(fit, plot_type = "ggplot")
plot(fit, parameter = "tau", conditional = TRUE,
      prior = TRUE, xlim = c(0, 1))
```




```
# specifying an informed one-sided hypothesis test
```

```
fit <- RoBMA(  
  d = Bem2011$d, se = Bem2011$se,  
  priors_effect = prior("normal", parameters = list(mean = 0, sd = 0.30), truncation = list(0, Inf))  
)
```

```
# specifying only a PET-PEESE style publication bias adjustment
```

```
fit <- RoBMA(  
  d = Bem2011$d, se = Bem2011$se,  
  priors_bias = list(  
    prior_PET("Cauchy", parameters = list(0,1), truncation = list(0, Inf), prior_weights = 1/2),  
    prior_PEESE("Cauchy", parameters = list(0,5), truncation = list(0, Inf), prior_weights = 1/2)  
  )  
)
```

```

fit <- RoBMA.reg(~ measure + age, data = df_reg)

summary(fit)

> Robust Bayesian meta-regression

> Components summary:
>
> Models Prior prob. Post. prob. Inclusion BF
> Effect      72/144      0.500      0.340 5.150000e-01
> Heterogeneity 72/144      0.500      1.000 1.043068e+23
> Bias        128/144      0.500      0.965 2.797600e+01

> Meta-regression components summary:
>
> Models Prior prob. Post. prob. Inclusion BF
> measure 72/144      0.500      0.950      18.940
> age     72/144      0.500      0.154      0.182

> Model-averaged estimates:
>
> Mean Median 0.025 0.975
> mu      0.063 0.000 0.000 0.330
> tau     0.213 0.209 0.149 0.301
> omega[0,0.025] 1.000 1.000 1.000 1.000
> omega[0.025,0.05] 1.000 1.000 1.000 1.000
> omega[0.05,0.5] 0.998 1.000 1.000 1.000
> omega[0.5,0.95] 0.997 1.000 1.000 1.000
> omega[0.95,0.975] 0.997 1.000 1.000 1.000
> omega[0.975,1] 0.997 1.000 1.000 1.000
> PET      2.043 2.484 0.000 3.277
> PEESE    1.012 0.000 0.000 9.811
> The estimates are summarized on the Cohen's d scale (priors were specified on the Cohen's d scale).
> (Estimated publication weights omega correspond to one-sided p-values.)

> Model-averaged meta-regression estimates:
>
> Mean Median 0.025 0.975
> intercept      0.063 0.000 0.000 0.330
> measure [dif: direct] -0.126 -0.129 -0.216 0.000
> measure [dif: informat] 0.126 0.129 0.000 0.216
> age            0.000 0.000 -0.047 0.047
> The estimates are summarized on the Cohen's d scale (priors were specified on the Cohen's d scale).

fit <- RoBMA.reg(~ measure + age, data = df_reg, priors = list(
  measure = prior_factor("normal", parameters = list(mean = 0, sd = 0.25), contrast = "treatment"),
  age     = prior("gamma", parameters = list(shape = 2, rate = 10))))

```

Advantages of RoBMA

- Can incorporate uncertainty about the selected model with BMA
- Can provide evidence for either the null or the alternative hypothesis
- Has better performance with small sample sizes
- Has the capacity to incorporate expert knowledge
- Has the potential for sequential updating of evidence

Disadvantages of RoBMA

- Slow - requires MCMC sampling ($2^p \times 36$ models)
- Can fail under strong p -hacking

Thank you for your Attention

R package: <https://cran.r-project.org/package=RoBMA>

JASP: <https://jasp-stats.org/>

For more about RoBMA:

Maier, M., Bartoš, F., & Wagenmakers, E. J. (2022). Robust Bayesian meta-analysis: Addressing publication bias with model-averaging. *Psychological Methods*.

Bartoš, F., Maier, M., Wagenmakers, E. J., Doucouliagos, H., & Stanley, T. D. (2023). Robust Bayesian meta-analysis: Model-averaging across complementary publication bias adjustment methods. *Research Synthesis Methods*

Bartoš, F., Maier, M., Stanley, T. D., & Wagenmakers, E. J. (2023). Robust Bayesian Meta-Regression—Model-Averaged Moderation Analysis in the Presence of Publication Bias. *PsyArXiv* <https://doi.org/10.31234/osf.io/98xb5>

